Faculty of Engineering

# Integer Linear Programming in Urban Renewal Development Projects (An Iraqi-Case Study)


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# Committee Decision

This Thesis "**Integer Linear Programming in Urban Renewal Development Projects (An Iraqi-Case Study)**" Was Successfully Defended and Approved on 11/06/2020

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# Dedication

**To ……………………………………..………… My Father & Mother**

**To ………………………………………..………… My Wife**

**To ………………………………………..………… My Flower (Son)**

**To …………………………………………………… My Brothers and Sisters**

**To …………………………………………………… Whom Likes My Succeed**

**Give Them My Achievement**

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# Table of Contents

# List of Figures

# List of Tables

**Integer Linear Programming in Urban Renewal Development Projects (An Iraqi Case Study)**

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## Abstract

Iraq has witnessed a series of wars and was subject to severe destruction, particularly after its invasion and occupation in 2003. In Light of the current economic situations in Iraq, optimal utilization of resources has become a pressing need.

This study aims at using Integer Linear Programming for mathematical model in urban development, taking constructing new houses in Iraq as a case study.

A mathematical model for the optimal distribution of sub-contractors to execute (170) proposed houses with different area (150,180, 200, 250, 300 $m^2$) and two execution levels (first degree and second degree) has been built, with the aim of minimizing costs and realizing the project within the planned time period.

Data was collected from the tenders of six sub-contractors, including unit price according to area and execution level, as well as the time period for unit accomplishment. Seven mathematical for seven case studies were used for sensitivity analysis. Through changing the available resources and requirements, the form of the constraints changed, where some decision variables and competition constraints were excluded and new constraints were added.

Integer Linear Programming was employed in solving the mathematical model equation with the aid of (Linprog) software. The results showed the possibility of

applying mathematical modeling and Integer Linear Programing effectively to achieve minimum execution cost within the planned time period, and optimum assignment of sub-contractor. Furthermore, the study finding revealed enabling the project manager or work owner to select the optimal solution according to the company's requirement.

The researcher recommends to expand using mathematical modeling for solving problems in construction projects.

**Keywords:** Construction projects, Mathematical modeling, Integer Linear programming, Optimization.

# List of Abbreviations

| **Abbreviation** | **Description** |
|---|---|
| ILP | Integer Linear Programming |
| LP | Linear Programming |
| DP | Dynamic Programming |
| UPP | Urban Planning Project |
| UDP | Urban Development Project |
| MILP | Mixed Integer Linear Programming |
| ERS | Emergency Response System |
| CHC2 | Community Health Center’s Service |
| SD | Second Degree |
| FD | First Degree |
| OR | Operation Research |
| ANN | Artificial Neural Network |

# 1 Chapter One: Introduction

## 1.1 Background

Financial resources were considered an important key in construction activities, but many projects were not executed according estimated cost and base line schedule. Because the process of construction project is very complex activity, market for construction project also complex and was required very carefully analysis researchers have constructed different model to develop finance-based schedule for multi-project such as El-Gafy (2007) was used ant colony optimization model to optimal resources for repetitive protectives, in order to minimization the duration. Abido, and Elazouni (2010) clarified how is the finance-based scheduling achieves the irrigation between the financing and scheduling objective, the authors were implemented pareto algorithm to advise optimum finance-based schedule of multiple projects. Urban renewable planning businesses is a high competition business in Iraqi construction project, the business was allocated in all cities after the war 2003.

Iraqi cities have witnessed tremendous destruction as a result of war over a long time, affecting all aspect of life and leaving people in need of basic services.

Improvement of life requires the availability of resources, without which projects and businesses are not likely to achieve success. This is particularly the case in construction Industry, where the environment is complex and dynamic and require cautious segment analysis, according to Taha (2017), urban development is generally concerned with three areas:

1. Building new housing facilities;
2. Upgrading existing housing facilities and recreational areas;
3. Planning public facilities, such as schools and health centers.

Such development projects face many economic as well as social challenges, such as the availability, of land for construction and financing resources, as well as creating innovative solutions to make the developed facilities appropriate to people with low income levies. There are some cost models that are used in construction project management to achieve appropriateness of construction activities to the financial budget allocated.

Accelerating some activities in construction projects can be carried out at additional cost to crash the project's implementation time. In such models balance must be considered between time and cost to avoid delays as well as cost overruns (Mohamed et al, 2008). In the Iraqi construction industry, urban development represents a sector with high competition.

This research has the objective to develop a mathematical model which can be utilized in urban project planning to minimize cost, taking both budget duration for each project into account.

## 1.2 Research Problem

A fundamental as result of war, numerous cities and villages in Iraq have been subjected to destruction. This situation called for providing urban development's projects to serve citizens whose were damaged, by constructing new housing facilities as well as planning public utilization facilities. This is not easy in light of limited resources. Innovative solution is required in managing the project budgets and executing the planned project within the shortest duration possible in a manner that suits the citizen's income levels. Subcontractors selected for these projects must be highly qualified and well experienced to ensure the success of the planned projects otherwise, these projects will suffer from implementation delays, cost over runs and poor quality.

## 1.3 Justification of the Research

1. Spread of weak implementation of construction projects, particularly in terms of cost, duration and quality.
2. Poor decision-making process in selecting the appropriate subcontractors for construction projects.
3. Urgent need for urban development projects in areas that have been severely affected by war in Iraq.

## 1.4 Research Objectives

The objectives of this research are summarized as follows:

1. Formulating mathematical model for analyzing how UDP including different types of houses are distributed among subcontractors.
2. Using integer Linear Programing to minimize the total cost for construction project execution.
3. Conducting sensitivity analysis to verify the proposed mathematical model.

## 1.5 Research Hypothesis

Below is the main hypothesis to be examined in this research, formulated as a null hypothesis and as an alternative hypothesis.

$H_0$: Mathematical models and optimization techniques cannot be used in urban planning project (UPP).

$H_{a:}$ Mathematical models and optimization techniques can be used in UPP.

## 1.6 Research Methodology

The methodology used in this research will include the following parts:

1. The theoretical part, which includes:

- Reviewing related literature to form a view about optimum project planning and resource allocation.
- Studying different types of mathematical models and optimization technique in order to select the most appropriate mathematical model and optimization technique that will help in an achieving the study's objectives.

2. The practical part, which includes:
   - A field survey of construction companies and selecting a large UPP in Iraq as the data source.
   - Creating a mathematical model using the data collected and solving it using the appropriate optimization technique, in order to identify how many units each subcontractor can execute within the cost and time predetermined in each house type.
   - Conducting sensitivity analysis by changing resource available to subcontractor constraints, type of constraints, budgets limitations, and duration required.

## 1.7 Thesis Structure

This thesis consists of five chapters as follows:

1. **Chapter One**: It includes a general introduction which consists of the research problem formulation, research motivation, research objectives, research hypothesis and research methodology.
2. **Chapter Two**: This chanter presents a detailed Literature review that describes previous work conducted in this field.

3. **Chapter Three**: It present the methodology adopted in this research and the optimization technique utilized.
4. **Chapter Four**: It consists of model formulation for the case study, sensitivity analysis and analysis of results.
5. **Chapter Five**: It presents the study's conclusions, recommendations and future research horizons.

# 2 Chapter Two: Literature Review

## 2.1 Introduction

Optimization techniques is among the widely used scientific method that assist in the decision-making process. It is considered a relatively modern applied science field that has success in numerous civil and military domains (Sharma, 2004).

Optimization techniques has numerous definitions, where society for operation research is considered among the most reliable ones. According to that definition, optimization techniques "the use of scientific methods to solve complicated problems in large_ scale systems of raw materials, machines, financial resources among others in civil or military institutions and companies.

The American society for operation de fined it as " the process of making piratical decisions in order to build and structure systems of equipment and human resources, within certain conditions that requires proper employment of rare resources (Taha, 2017).

Optimization technique is generally the use of mathematical ways and techniques depending on the relationships among variables in equation with the aim of obtaining ideal results in terms of maximizing profits and minimizing expenses.

According to Matousek, et al. (2017), the main pillars to optimization technique can be summarized as follows:

1. Using the scientific method in research, including recording results, measuring and identifying variables models to represent the phenomena under investigation in addition to hypothesis formulation and testing to obtain results.
2. Using the holistic approach through studying the relevant phenomena from all view points and analyzing them to their different components.

3. Using a variety of experiences and background, to which is best achieved through forming coordinating team of researchers with relevant major and background process the investigated phenomena from all points.
4. Using mathematical models which are created with the aim of analyzing the problem and finding proper solutions.
5. Using information technology, including data collection, organization, analysis and processing, such processes require the usage of computer programs.

## 2.2 Previous Studies

Optimization is used in various fields utilizing several techniques. There are many published studies in this aspect. Below is a brief summary of some of them.

Fatma A. Agrama (2012) was studied a genetic algorithm used to carry out optimized scheduling of linear construction project.

Linear construction project with numerous identical units, in which activities are repeated. The model proposed enables construction planner to create optimal/near-optimal plans to minimize project duration, work delay and interruptions number of labors. An example was analyzed using the proposed model.

EL. Abbasy, et al (2015), in his study was conducted with the aim of minimizing duration, cost and negative cumulative balance in multiple mode construction projects using a sorting genetic algorithm to address the issues of critical path method (CPM) scheduling and project cash flow. Through application of the proposed model to a hypothetical example, it was revealed that the proposed model was successful in achieving a trade-off among the three factors mentioned above in multiple mode construction projects.

kane and Tissier.(2012) , explained a model consisting of two parts was proposed to assist decision makers in accelerating their projects and optimizing resource allocation to lower the cost of execution of a number of an associated projects. Three projects were taken as case studies to be implemented by using the proposed model, were the results were promising in terms of speeding up project as well as cost optimization.

EL- Kholy, A (2014) used a multi-objective fuzzy LP model for maximizing final cash balance, minimizing total cost and minimizing initial capital, nothing that these objectives seem to be contradictory to each other. In the proposed model, the impacts of advanced payment, progressive client payment and penalty of delayed payment where considered in the proposed model. A relationship was found that achieves a type of balance among the study objectives mentioned above. The proposed model was tested, where the results showed that it is characterized by simplicity as well as by extendibility to include many more objectives to realize a trade-off relationship among them.

Hamzeh H. et al (2018) was used LP technique to present optimal crashing time scheduling in mega projects consisting of two phases. The mega project is first analyzed to identify the optimal crashing time from the accomplishment of each of the critical milestones in the project whereas in the second phase, the obtained crashing time is distributed over the mile stones of the mega project, the methodology presented in this study enables project managers to adjust crash time-cost trade-off.

Abido, and Elazouni (2010) investigated the use of integration between scheduling and financing aspects in projects by utilizing the so-called finance-based scheduling.

The proposed method uses am algorithm to achieve finance-based scheduling of simultaneously executed construction projects under cash constraints, taking into account the minimization of execution Period, required credit and financing cost of a group of projects.

Abunada and Mohammed (2018). We're identifying and ranking the factors considered by the main contractors in the process of selecting subcontractor in Jordan. Bid price execution time and pre-qualification have been found to influence the subcontractor selection process. The study found that distribution of bids among sub-contractors is deferent when the pre-qualification is used and the total cost of all project will differ too, the study was used assignment technique.

Gadge, et al (2014) used optimization in land and water resource management when using surface irrigation. The aim of this study was to find out the optimal net profit passible using (LP) as an optimal method. The only restrictions were water and land. The researchers formulated a linear model that can be referred to whenever changes occur to restrictions. This study was an attempt to obtain the optimal agricultural pattern that yields the best net profit possible.

In transportation field, Mohamed et al. (2008), applied LP in multiple stages of transportation. Sensitivity analysis was conducted to find out the optimal qualities at minimum cost. Similarly, Rajelyan et al. (2013) used LP to solve the problem of high transportation cost in companies. The researcher revealed the positive role of (LP) in solving the problem, where the proposed model applied by marketing and executive managers with high efficiency to minimize cost and maximize performance. LP has been employed in product management, a marketing task and company stock control.

In the field of investment, Gherghina (2013), used (LP) optimization to achieve maximum profit by enhancing two labs. This research came due to recommendation that is not feasible to use resources to enhance one lab only. It was concluded that resources should be used to enhance two Labs to maximize profit.

AZimi et al. (2013) presented an idea to achieve optimal resource management. The research was based on creating a structure of newly established companies using (LP), to arrive at the optimal human resource management, as decision making an affect the success of companies. The researchers found that companies nowadays are unable to manage institutions and take responsibilities in current competitive global market which emphasizes the importance of having a model of optimal management of human resources. The use of (LP) in optimization kept expanding and covered numerous production area.

Ezema and Amakom (2012), used LP to increase the profit of a plastic factory. The researchers found that two of the raw materials were of excessive quantities while the rest were scarce. Another finding was that the company investigated had to use consultants from outside the company to help company make optimal decisions.

In water engineering, Ghaderi et al (20l0), used mixed (ILP) to build an optimal water tank model. This paper did not focus on the economic cost but rather on the optimal operation of water tank. The researchers found that the result of (MILP) out performed those of (LP). The application of (MILP) decreased over-flooding and increased the amount of water utilized.

Chen and Shahandahti (2007), simulated annealing algorithms for optimizing multi-project linear scheduling with multiple resource constraints. They used a simulated annealing algorithm model for finding the best sequence of activities to minimize project duration and resource usage fluctuations. They argued multi-projects can be scheduled using either the multi - projects approach or single- project approach. In this study the multi – project. Approach was used, where two projects were solved using the proposed model and the results written in Java programming language and the program run for (100) times then the result of implementation, this program was presented without comparing it with any previous result.

Abbas and Moula (2010) used two new algorithms to solve (MILP) problem utilizing the cutting plane method , Likewise, Nassar (2011) used genetic algorithms to optimally assign resources for repetitive construction projects aiming at minimizing the overall project duration as well as minimizing interruptions the and early finish of the activities were determined. In the second part, late start and finish date were calculated, while in third part, the overall project finish date was calculated taking interruption days into account. The results of proposed model were compared with those of similar models.

Shah et al (l993) was used (LP) to obtain optimal settlement of resources, nothing that project managers often face many options for scheduling the project resources. The researchers developed a computer program emphasized to use available resources as an input and obtain the optimal resource settlement as an output.

Elazouni and Gab-Allah (2004) introduced an (ILP) based on the finance basics in order to produce a schedule balancing between available cash and financing required for activities at any period throughout duration of project. Minimizing model was formulated to support an (ILP) model subjected to many constraints, activity shifting, sequence of activity and specified credit limitation. The model determined the activities that minimize the duration of project model implemented on small project offered a realistic and useful scheduling concept.

Zheng et al (2004) proposed a multi-objective modified adaptive weight approach model and compared it with single-objective model propose d by (Hegazy 2002). It was shown that non- replaceable points could correctly be located by the new model on the pare to diagram, bearing the limitation of cost and time in mind. It was revealed that the model provides manager with more realistic flexibility in analyzing their decisions. In order to simulate uncertainty, zhang, zand zheng (2004) used fuzzy-sets theory and produced better results, especially in high risk cases. However, the number of solutions generated for decision making was noticeably decreased and necessary adjustment were proposed for improving the efficiency of the model when applied to larger and more complicated projects.

Toro- Diaz et al (2013) used a myopic allocation policy which produced an optimal solution to minimize long-run average cost (response time) of ambulance converge.

The researchers developed a mathematical formulation integrating an (ILP) model with a hypercube model. The aim of this research was to minimize response time and maximize coverage, where two alternative criteria were considered in this research; namely, variability of individual response time and variability of ambulance workloads.

Likely Billhardt et.al (2014) proposed maximizing the converge of demand points by more than one ambulance or utilizing double standards to achieve the best coverage. The aim of this research was to improve the response time defined as the time period between a patient call and the moment at which an ambulance arrives and the patient can receive medical assistance, which is one of the essential performance indicators. Two mechanisms were proposed to achieve effective ambulance coordination; namely, dynamic ambulance allocation mechanism and dynamic ambulance redeployment mechanism, the researchers evaluated their model by simulating a real-world scenario.

Basu.et al (2017) proposed a new frame work that is capable of esteeming existing emergency response system (ERS) presenting solutions to development and adding facilities at optimal location. In term of system performance metrics, such as; first response time. Area coverage and average response time. The researchers compared (ERs) performance before and after implementation of the proposed framework to determine the degree of improvement in terms of achieving maximum coverage through minimum resource allocation.

Griffin et al (2008) used optimization in resource allocation by presenting optimization model to set the location of community health center (CHC) and the services provided by each (CHC) in a geographical network. The state of Georgia was chosen as a prototype; then the researchers moved to entire united states. This model showed improvements in all measures and the results revealed that all of the locations services and capacities should be determined.

In large-scale emergencies, different models were presented by Jia, et al (2007), who proposed a number of models to address problems related to emergency situation and facility location covering model, a P-median model and a P-center model were offered. The proposed models helped in reducing life and economic lasses compared to traditional models. The study aimed at facility locations which are useful and helpful.

In the state of large- scale emergences offering tailored location models that are suitable to the characteristics of and large- scale emergencies Meinzer and storandt (2014) introduced several strategies to carefully position ambulances in their service range and determine sensible mappings of ambulances to patient requests. They compared those strategies in term of number of patients saved and analyzed them from un economical perspective the allocation and redeployment strategies discussed in this research were as follows:

- **Greedy Agent:** This strategy aims at always sending the closet ambulance to new patient request.
- **K-Method Assignment:** Here, the aim is to minimize the average distance of each possible request origin to the nearest ambulance thus minimizing the response time to the next request.
- **Vorani-based Allocation:** In this strategy free ambulances are only placed at hospitals bases; or therefore, the researchers divided the graph into cells resulting in Voronoi- diagram with hospitals as seed K.Medoid Assignment and Voronoi - based allocation outperformed the conventional greedy agent approach in several scenarios as experiments showed on the other hand, three categories of diverse static ambulance location models were presented by (Belanger et al., 2015) - They reviewed p previous works on statics ambulance location problem s and provided new solution and new models. In this research diverse static ambulance location models were divided into three categories:
    1. Single converge deterministic models using the notion converge, meaning that if each demand zone can be reached by at least one ambulance within, a prescribed time or distance frame, the demand zone is said to be covered.
    2. Multiple converge deterministic models based one increasing the number of ambulances to be available to cover a demand zone.
    3. Probabilistic and stochastic models having been developed to provide a realistic representation of real-life situations.

ELGohary, et al. (2017) presented an engineering concept to document, control, predict, and improve the contractor's labor productivity. The proposed engineering approach was applied to model construction labor of productivity of two construction crafts, carpentry and fixing reinforcing steel bars of different type of concrete foundation for residential and commercial building using the technique of Artificial Neural Network (ANN) to map and quantify the relationship between the influencing factors and the corresponding productivity rate and utilizing transfer function of the hyperbolic tan function. The result showed that the network adequately converged and have noticeable and reasonable generalizing capabilities.

## 2.3 Modeling Types

Models are classified as follow (Mohamed, 2008):

### 2.3.1 Physical Models

Physical models are considered as miniature replications of the original case. A physical model is characterized to be effort and time consuming. It usually aims at analyzing the behavior of a specific existing system, or finding the best design possible for a system still to be constructed.

### 2.3.2 Empirical Models

Empirical model refers to activities related to creating model by observation and experiment. Empirical modeling can be considered computer- based. This conception of empirical is because modeling has been tightly associated with thinking about the role of computers in the model-building process.

### 2.3.3 Mathematical Models

Mathematical modeling has the aim of obtaining the best possible using mathematical theories and formulae. The outcomes of mathematical models are accompanied by limitation and hypotheses (Pual. H, 2009). Computer software can be utilized for achieving time efficiency model consist of a number of formulae A mathematical that might be differential or of other types depending on the nature of the system- studied, mathematical model can be classified according to (Liberti, 2010) as follows:

#### 2.3.3.1 Mathematical Model Based on the Level of Verification

1. Deterministic models which have to be completely confirmed in detail such as those related to price and production methods. Each factor in these models has a certain out came examples of the methods used to build such models are, linear and nonlinear programming and business networks.
2. Probabilistic models which cannot be fully predicted and include a certain degree of uncertainty. Every factor in these models has more than one potential outcome with a given degree of probability. Probabilistic models associated with random error.

#### 2.3.3.2 Mathematical Model Based on the Mathematical Form

According mathematical form can be classified as follows:

1. Linear models formulated from linear formula where all formula is of the first degree and graphs straight line.
2. Non-Linear mode ls fully or partially formulated non- linear formula, such as squared or logarithmic formula.

#### 2.3.3.3 Mathematical Modeling Stage

To formulate a mathematical model, the following Steps are to be followed. (Basu et al 2017).

1. Problem specification in this step the problem is closely studied from all perspective factors affecting the problem, circumstances around it perspectives and all its components.
2. Goal determination is identified integrated and intersecting goals, since some of the goals might be contradictory in nature.
3. Data collection: This step differs from one case to another according to the approach that best suits the goals previously set.
4. Specification of variables and available resources. When building model, it is the type of variables used, whether being internal or external variables. In addition, it is important to identify the resources available.
5. Mathematical importance to focus on building the model, which means the actual formulation of the appropriate model according to the nature of the problem under study and the factors affecting it.
6. Solving the model by using suitable optimization technique. Nowadays this is done with the help of specialized computer software in order to save time and effort.
7. Sensitivity analysis. In this step, it is required to compare the result obtained would get in reality this enables modifying the model to with those we get more realistic.

## 2.4 Optimization Techniques

Optimization may be defined as the processes by which an optimum is achieved. The optimum may be that of an industrial institution or an objectives function that the model a similar entry. The factor that define the optimum will vary with the situation to which the optimization process is applied. There are many techniques for solving optimization problem such as these (Billhard, et al. 2014).

### 2.4.1 Dynamic Programing

DP is a mathematical technique used to solve complex problems with multiple variables. It is utilized to find optimal solutions particularly multi- stages project that require a series of connected decision usually, the so- called back ward induction method is used in DP.

### 2.4.2 Nonlinear Programing

This technique is similar to (LP) technique in terms of reliance upon resource distribution and usage, but with the difference that nonlinear programing, the formulae used are nonlinear. External value finding in this technique can be classified into:

1. Finding the external value of a non-restricted value.
2. Finding the external value of a restricted value.

### 2.4.3 Inventory Adjustment Method

This is an essential optimization that achieved success in reducing budget in industrial, service, and business units, as a result of the importance of inventory factor.

### 2.4.4 Linear Programing

LP is an essential optimization an essential technique where was using in complex relationship that linear for mule. The relationship may be much more complex, so we simplify, to linear for mule.  It is optimally used in cases of scarce resources and limited possibilities, getting the result possible. The main aspects of LP are as follow, (Dantzig et al 2016).

1. Optimal distribution, as resources should not be randomly distributed since this would affect significantly affect the budget.

2. Available resources, which should be kept in mind while making decisions. Resources might be financial raw material, machines, working hours, ---etc.
3. Different uses, where there are various substitutions to the uses of optimization.
4. LP can be solved by graphic method when variables not more than two.

Simplex method which is an important analytical method than can be used to a variety of (LP) problems regardless of the number of variables and, dual simplex method.

## 2.5 Integer Linear Programming

This approach relies on ignoring integer number, then dealing with them after blaming final result. There are two main types of (ILP) based on decision variables (Matosek, 2017).

- Pure integer program when all decision variables are integer.
- Mixed integer program when some, but not all variables are integer.

The general form of ILP optimization problem is:

$$Max\ or\ Min\ Z = \sum_{i=1}^{n} C_i y_i \qquad \textit{Equation 2.1}$$

Subjected to

$$a_{11}y_1 + a_{12}y_2 + \cdots + a_{1n}y_n\ (\leq, =, \geq)\ b_1 \qquad \textit{Equation 2.2}$$

$$a_{21}y_1 + a_{22}y_2 + \cdots + a_{2n}y_n\ (\leq, =, \geq)\ b_1 \qquad \textit{Equation 2.3}$$

$$a_{m1}y_1 + a_{m2}y_n + \cdots + a_{mn}y_n\ (\leq, =, \geq)\ b_m \qquad \textit{Equation 2.m}$$

$$y_i \geq 0\ and\ integer \qquad \textit{Equation 2.4}$$

And in matrix terms is

$$Max\ or\ Min\ Z\ =\ C^T\bar{y} \qquad \textit{Equation 2.5}$$

Subject to

$$A\ \bar{y} \geq, =, \leq b \qquad \textit{Equation 2.6}$$

$$\bar{y} \geq 0\ and\ integer \qquad \textit{Equation 2.7}$$

Where:

$y_1, y_2, \ldots, y_n$: decision variables

$C_i$: cost or benefits

$b_1, b_2, \ldots, b_n$: resource available

### 2.5.1 Integer Linear Programming Hypotheses

When (ILP) is used in business, it is deemed a mathematical method of resource management. (ILP) relies on hypotheses in order to be used. These hypotheses can be summarized as follows (Dantzig, 2016):

1. Linearity, meaning that the relationship among the variables of the problem are based on fixed ratios. Linearity is looked at two perspectives:
    - Mathematical linearity, which refers to the use of linear formula.
    - Economic linearity, which reflects the proportions between incomes and outcomes.
2. Certainty, meaning assuming that the future is certainly, all formulae should have outcomes with no degree of doubt.

3. Proportionality, meaning that every action is independent by itself, so that the objective function is the summation of different outcomes. Quantities related to different resources are proportional to needs of these resources.
4. Non negativity, meaning that all variables outcome must be positive.
5. All variables must be integer number.

### 2.5.2 Types of Possible Solution in ILP and LP

When solving a linear programing problem, two regions have been found:

1. Impossible solution region. It is that region outside of feasible region and does not meet the constraints of the problem.
2. Feasible region. It is region consisting of the group of values that meet the constraints of problem and including the following solution:
   - Acceptable solution, which are all solution that meet the problem's given constraints.
   - Main acceptable solutions, which represent the extreme solution, one of which is the objective function.
   - Optimal solution, which represents the final outcome, the maximum or minimum value according to the problem (Genova and Guliashki, 2011).

### 2.5.3 Special Cases in ILP and LP

Special cases in (ILP) and (LP) can be summarized as follows:

1. Infeasible solution. This case is encountered when the problem investigated has contradicting constraints. In this case, it would be impossible to find feasible region, meaning that the problem no solution.

2. Degeneracy. This case occurs when the problem studied contain a redundant constraint, which is a constraint that can be removed without affecting the feasible region. However, a redundant constraint increases the number of steps that should be carried out to find the optimal solution.
3. Unbounded case. This case is faced when the solution region is open from one side. This means when cannot determine the optimal solution. From an economic perspective, this scenario is very unlikely to happen, as there is no company with unlimited resource. The occurrence of this case is an indication of some flaws in the program that must be fixed.
4. Alternative optimal solution. This case appears when the problem studied has more than one optimal solution, leading to some earnings the same cost in the case of minimization. (Dantzig, 2016).

### 2.5.4 Integer Linear Programming Approach

Some of the most commonly used method of (ILP) as follows, (Genova and Guliashki, 2001).

#### 2.5.4.1 Cutting – Plane Methods

The concept of cutting-plane method is very simple for a linear problem. LP problems are the same as ILP problem, but without the constraint variables integer values. If the solution was an integer value, it would be exactly the same as the ILP solution without adding any constraint. Constraint is added in cases where all outcomes of the linear program are valid solution for the integer linear program, while all optimal fractional outcomes are excluded. Theoretically the "cutting" of solutions is carried out by adding one constraint at a time and repeating that until obtained an integer optimal outcome. We can further explain this method through the following steps:

1. Start to solve the given problem as simplex LP, ignoring the integer constraint on variables. If the final outcomes have integer values, stop here. But, if the final outcomes were fractions, move to the next step.

2. From the final outcomes, take the variables with the largest fractions which reduce repetitions and time spent in rounding the outcomes. Then, generate cutting-plane constraints.

3. To generate the cutting-planes constraints, assume that the outcomes were the following formula:

$XB_i + \sum j \left(a_{ij}\, x_j\right) = b_j\,,\ j \in 1$ *Equation 2.8*

$XB_i + \sum j \left(\left[a_{ij}\right] + f_{ij}\right) x_j = b_i + f_i$ *Equation 2.9*

$XB_i + \sum j \left(\left[a_{ij}\right] x_i\right) - \left[b_j\right] = f_i - \sum j\, f_{ij} x_j \leq \mathbf{0}$ *Equation 2.10*

The new constraint:

$f_i - \sum j\left(f_{ij}\, x_j + s\right) = 0$ *Equation 2.11*

Where: $\left[a_{ij}\right] - a_{ij} = f_{ij}$ is the fractional part $0 \leq f_{ij} < 1, a_{ij}$

$\left[b_i\right] - b_i = f_1$ is the fractional part $0 \leq f_i < 1, b$

S is a new possible integer dissolved variable.

4. Test the optimal plane cut (here, we test the outcomes of step 3) in order to get the optimal cut with the minimum number of steps. That is carried out through the substitution of the values of inactive variables that we get from the constraints of the original problem with the constraints of the cutting-plane

outcomes. The results of this substitution are lines that intersect with the possible solutions area. Choose the line that intersects with the largest region.

5. Add the new constraint obtained from step 4 in the final table of the simplex method, then solve the problem as a linear programming problem. If the outcomes were integer values, then the solution is over if not, move back to step2.

To better explain the method let's take the following example:

$Max\ Z = 5x_1 + 8x_2$ *Equation 2.12*

Subject to:

$x_1 + x_2 \leq 6$ *Equation 2.13*

$5x_1 + 9x_2 \leq 45$ *Equation 2.14*

$x_1, x_2 \geq 0$ *Equation 2.15*

Solution:

Table (2-1) represents the optimal table for the solutions of the linear programming problem without the integer constraint.

Table 2-1: Optimal Table of Formula, for the Simplex Method

| **Basic Variables** | $\boldsymbol{X_1}$ | $\boldsymbol{X_2}$ | $\boldsymbol{S_1}$ | $\boldsymbol{S_2}$ | **R.H.S** |
|---|---|---|---|---|---|
| P | 0 | 0 | $\frac{5}{4}$ | $\frac{3}{4}$ | $\frac{165}{4}$ |
| $X_1$ | 1 | 0 | $\frac{9}{4}$ | $\frac{-1}{4}$ | $\frac{9}{4}$ |
| $X_2$ | 0 | 1 | $\frac{-5}{4}$ | $\frac{1}{4}$ | $\frac{15}{4}$ |
| $\boldsymbol{X_1 = \frac{9}{4}}$ | | $\boldsymbol{X_2 = \frac{15}{4}}$ | | $\boldsymbol{Z = 41\frac{1}{4}}$ | |

Using the cutting-plane method, we get:

$$-Z = \frac{5}{4} s_1 - \frac{3}{4} s_2 = 41\frac{1}{4} \qquad \textit{Equation 2.16}$$

$$x_1 + \frac{9}{4} s_1 - \frac{1}{4} s_2 = \frac{9}{4} \qquad \textit{Equation 2.17}$$

$$x_2 - \frac{5}{4} s_1 + \frac{1}{4} s_2 = \frac{15}{4} \qquad \textit{Equation 2.18}$$

$$x_1, x_2, s_1, s_2 \geq 0 \qquad \textit{Equation 2.19}$$

$$-Z = -2 s_1 - s_2 - 42 = \frac{31}{4} - \frac{3}{4} s_1 - \frac{1}{4} s_2 \qquad \textit{Equation 2.20}$$

$$x_1 + 2s_2 - 2 = \frac{1}{4} - \frac{1}{4} s_1 - \frac{3}{4} s_2 \qquad \textit{Equation 2.21}$$

$$x_2 - 2s_1 - 3 = \frac{3}{4} - \frac{3}{4} s_1 - \frac{1}{4} s_2 \qquad \textit{Equation 2.22}$$

$$\frac{3}{4} - \frac{3}{4} s_1 - \frac{1}{4} s_2 \leq 0 \qquad \textit{Equation 2.23}$$

$$\frac{3}{4} - \frac{3}{4} s_1 - \frac{1}{4} s_2 \leq 0 \qquad \textit{Equation 2.24}$$

Testing the optimal plane cut out of the previous plan-cuts:

$$\frac{3}{4} - \frac{3}{4} s_1 - \frac{1}{4} s_2 + s_3 = 0 \qquad \textit{Equation 2.25}$$

$$\frac{1}{4} - \frac{1}{4} s_1 - \frac{3}{4} s_2 + s_4 = 0 \qquad \textit{Equation 2.26}$$

From previous equations, we get $(s_1, s_2)$ as follows:

$$s_1 = 6 - x_1 - x_2 \qquad \textit{Equation 2.27}$$

$$s_2 = 45 - 5x_1 - 9x_2 \qquad \textit{Equation 2.28}$$

We substitute equation (2-20) and (2-21) in the first and the second planes in order to get the main planes of the problem, given in the form of variables in the problem.

$2x_1 - 3x_2 \leq 15$ *Equation 2.29*

$4x_1 - 7x_2 \leq 35$ *Equation 2.30*

We test the optimal plane cut by substituting the increase or decrease of the values of $(X_1, X_2)$ in the plane cuts as follow:

$\frac{-1}{4}2 + \frac{-3}{4}3 = -2\frac{3}{4}$ *Equation 2.31*

$\frac{-1}{4}4 + \frac{-3}{4}7 = -6\frac{1}{4}$ *Equation 2.32*

Since the optimal plane cut is the first one, we use it to solve the equation

$\frac{3}{4} - \frac{3}{4}s_1 - \frac{1}{4}s_2 \leq 0$ *Equation 2.33*

$\frac{3}{4} - \frac{3}{4}s_1 - \frac{1}{4}s_2 + s_3 \leq 0, s_3 \geq 0$ *Equation 2.34*

$-\frac{3}{4}s_1 - \frac{1}{4}s_2 + s_3 = -\frac{3}{4}$ *Equation 2.35*

Table 2-2: Table of the Linear Integer Programming Method

| **Basic Variables** | $X_1$ | $X_2$ | $S_1$ | $S_2$ | $S_3$ | **R.H.S** |
|---|---|---|---|---|---|---|
| P | 0 | 0 | $\frac{5}{4}$ | $\frac{3}{4}$ | 0 | $\frac{165}{4}$ |
| $X_1$ | 1 | 0 | $\frac{9}{4}$ | $\frac{-1}{4}$ | 0 | $\frac{9}{4}$ |
| $X_2$ | 0 | 1 | $\frac{-5}{4}$ | $\frac{1}{4}$ | 0 | $\frac{15}{4}$ |
| $S_3$ | 0 | 0 | $\frac{-3}{4}$ | $\frac{-1}{4}$ | 1 | $\frac{-3}{4}$ |

Table 2-3:Optimal Solution of the Linear Integer Programming Method

| **Basic Variables** | $\boldsymbol{X_1}$ | $\boldsymbol{X_2}$ | $\boldsymbol{S_1}$ | $\boldsymbol{S_2}$ | $\boldsymbol{S_3}$ | **R.H.S** |
|---|---|---|---|---|---|---|
| P | 0 | 1 | 0 | $\frac{-1}{3}$ | $\frac{-5}{3}$ | 40 |
| $X_1$ | 1 | 0 | 0 | -1 | 3 | 0 |
| $X_2$ | 0 | 1 | 0 | $\frac{2}{3}$ | $\frac{-5}{3}$ | 5 |
| $S_3$ | 0 | 0 | 1 | $\frac{1}{3}$ | $\frac{-3}{4}$ | 1 |
| $\boldsymbol{X_1 = 0}$ | | $\boldsymbol{X_2 = 5}$ | | | $\boldsymbol{Z = 40}$ | |

## 2.6 Branch and Bound Method

It is used for those problem having two variables only, the following steps followed to solve by branch and bound method (Taha, 2017).

- Step1: using graphical method to solve two variable problems, with ignoring the integer variable required.
- Step2: if the solution obtained does not have any fractional value that mean it's an optimal solution. Otherwise this called non- integer optimal solution and it should have been transformed to integer.
- Step3: put the obtained solution as a main node with the initial upper bound then create new two nodes should have two additional new constraints, the fractional value is between these constraints.
- Step4: use graphical method to find the solution for all nodes, if all variables have integers in node 1 and it is bound lower than the other node 2; can reached to the optimal solution already. If not go to next step.

- Step5: put new two nodes from that node still have fractional value. Repeat the same procedure to get the optimal solution.

## 2.7 Sensitivity Analysis

Sensitivity analysis is extremely important to decision makers, as a result of the dynamic environment that we live in. Required raw material prices and technologies available are subjected to changes. In order to arrive at the optimal solution in (ILP), raw material prices, product-fixed values and available resources should be considered. In reality product marketing could face difficulties in terms of economic situations, evolving technologies, recession, sale ability, -- etc. In such cases, the optimal solution would change, calling for the need to resolve the problem using the new values to obtain the new optimal solution. The question here, can we find the optimal solution without resolving the problem? The answer lies in sensitivity analysis. Decision makers need to know how probable changes might affect the optimal out comes. Some of those changes can be e clarified as follows (Rajeiyan, et al. 2013).

1. Changes in the objective function. Objective function usually contains variables representing earning and profit requiring maximization or losses and expenses requiring minimization. The decision maker might want to know the sequence of desired changes in any of those variables. Changes in the objective function variables might occur in the unessential variables coefficient unessential variables are those not included in the optimal solution table. Any change to the coefficient of an unessential variable will only affect the variable itself without influencing any other variables in the model. Other changes in the essential variable’s coefficient. The variables appearing in the final optimal solution table.

2. Change in the factor affecting the problem, the changes are needs might occur for multiple reasons such as:
    - The use of advanced technologies;
    - The evolution of skill due to training and experience;
    - Damage control application;
    - Reduction of production time;
    - Alternating one or more of the resources.
3. Addition of new variables and constraints or removing some variables or constraints.
4. Change the resource availability.

## 2.8 Summary of the Literature Review

This section will show the major previous work conducted in the field of using optimization technique in engineering management.

1. From pervious studies, some authors used genetic algorithm to minimize duration, cost and work delay, also interruptions number of labors.
2. Some other authors used LP to reduce cost, payments, delays, and crashing time scheduling in mega projects, also used LP to increase the profit of a plastic factory.
3. Some others used dynamic programming for dams management.
4. Researchers used neutral network to find productivity of labors in some construction activities.
5. Some researchers used non-linear programming to optimize the minimum water treatment plant.

6. In water engineering some researchers used mixed Integer Linear Programming to build an optimal water tank model, the application of (MILP) decreased over-flooding and increased the amount of water utilized.
7. The present study will use Integer Linear Programming to assignment the sub-contractor in urban renewal planning in order to minimize the total cost with limited duration.

# 3 Chapter Three: Research Methodology

## 3.1 Introduction

There is a problem which faces countries that have witnessed wars and severe infrastructure destruction. This problem lies in the necessity of replanting and developing harmed cities as well as building new house to substitute destructed ones. The constraints imposed on such project are usually of economic nature (land; construction; financing). Therefore, it is necessary to choose from among multiple alternatives, so that the houses implemented serve all society categories according to their personal incomes. These alternatives include houses with different areas and implementation methods and thus with different prices. These projects are also required to be accomplished timely and with the least possible costs, which means that it is required to conduct a process of contractor selection for implementing these projects in an optimal manner and within the least possible budgets. This was the basis of the idea of this research.

## 3.2 Study Area

Iraq has suffered from server destruction as a result of wars, which caused the destruction of the infrastructures in many Iraqi cities as well as extensive immigration waves of many Iraqi families. This has motivated numerous construction companies to think about finding solutions to this pressing problem. An Iraqi construction company proposed the implementation of (170) living houses with different areas and implementation classes to serve at the harmed citizens in the city.  The study location has been selected in Al-Anbar city in the western part of Iraq. The project site is located on the external road that connects with the western districts in the Al-Anbar city. Figure (3-1) illustrates the study area.

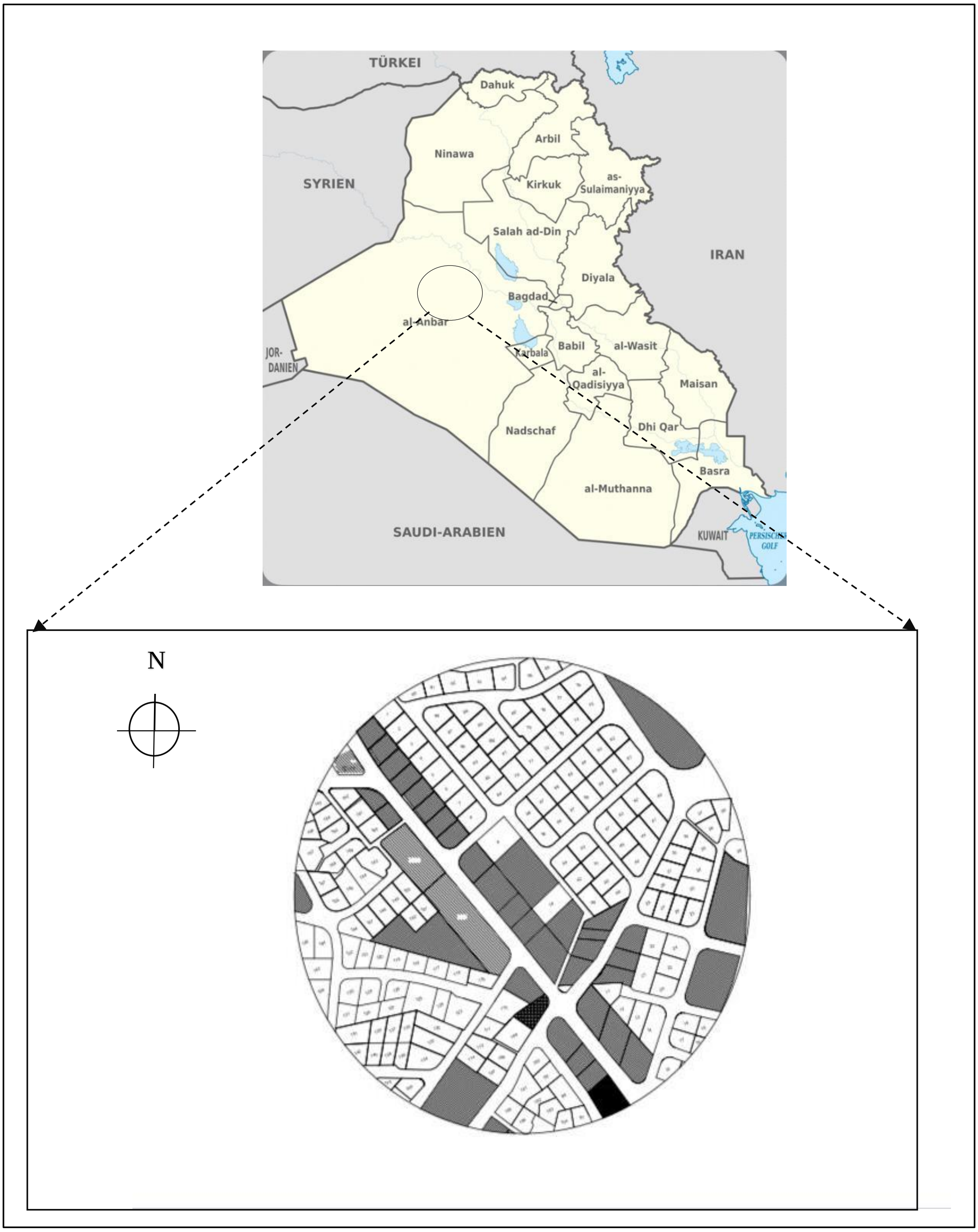


Figure 3-1: Geographic Map, and Plan of Proposed House Project

## 3.3 Case Study

The study proposal, presented by an Iraqi company (x) encompasses the implementation of a number of living houses to serve the citizens who have lost their houses. The proposed houses are of different types in terms of area and implementation type to serve most categories of citizens whose houses have been destroyed. The proposed houses have the following specifications:

1. (170) houses with different areas (150, 180, 200, 250 and 300 m).
2. Two implementation types according to primary materials and implementation method. Two methods have been chosen (first implementation class and second implementation class).
3. The time period of project accomplishment is determined by the company.
4. For the purpose of implementation, the company proposed referring the project to a number of sub-contractors after submitting their offers.
5. In order to distribute the houses to be implemented over the qualified Sub-contractors, a study should be conducted; through mathematical modeling and linear programming. Figure (3-2) illustrates the main steps of the research methodology.

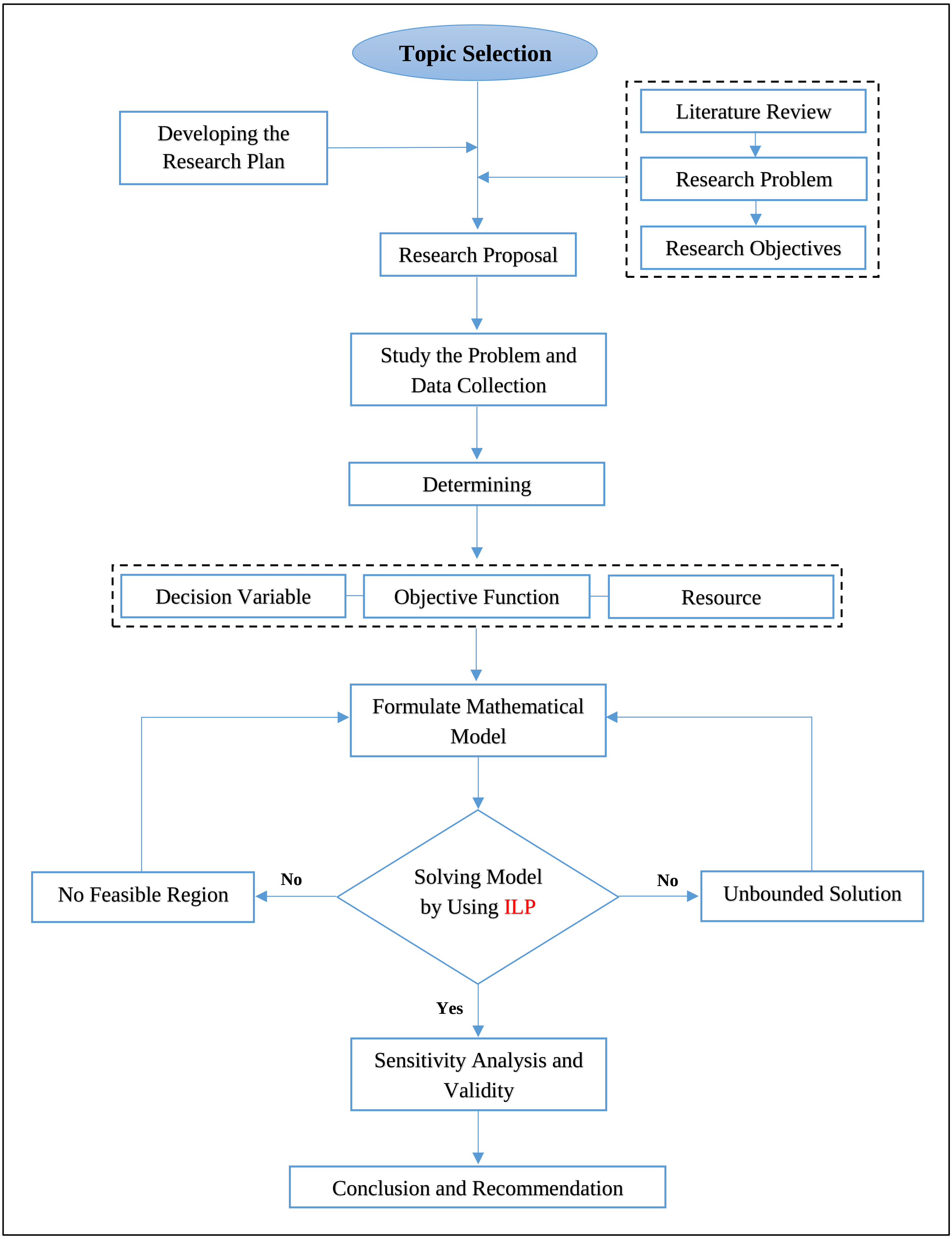


Figure 3-2: Research Methodology

## 3.4 Data Collection

The data collection process differs according to the problem under investigation. After identifying the aim of the study, it becomes clear what kind of data and information should be collected, as well as the variables associated with all dimensions, factors and circumstances surrounding the problem. The study variables must be distinguished from each other and the factors affecting the research problem must be divided into quantitative factors and quantitative ones. After identifying the data type, the data sources must be determined. The data obtained according to data source can be dividing into:

- Data obtained from companies as well as from different governmental and non-governmental entities.
- Data obtained through field visits, field survey and questionnaires.

In our study, data is obtained from the project proposing company and the tenders of the sub-contractors.

## 3.5 Mathematical Model

Mathematical modeling is a process of representing the actual problem in the simplest possible form, where the mathematical model consists of a set of mathematical equations (≥ ،= ،≤) that are connected together with a target function which is clear and simplified to represent the actual problem. El-Rayes and Moselhi; defined the mathematical model as the group of logical relationships, whether being qualitative or quantitative, that are associated with the phenomena tightly connected to the actual problem. In order to build a mathematical model, the following conditions are required.

1. Identification of the decision variables associated with each alternative, which are connected to a cost or a return. In our research, these are represented in the area and implementation method of the houses to be built.

2. The existence of a target to be achieved by the model, which can be minimization or maximization.
3. The availability of alternatives to choose from among in mathematical model construction.
4. Identification of available resources. Mathematical models can be deterministic models or potential models based on the certainty degree and can be linear or nonlinear according to the mathematical form. Figure (3-3) illustrates the stages of mathematical model construction.

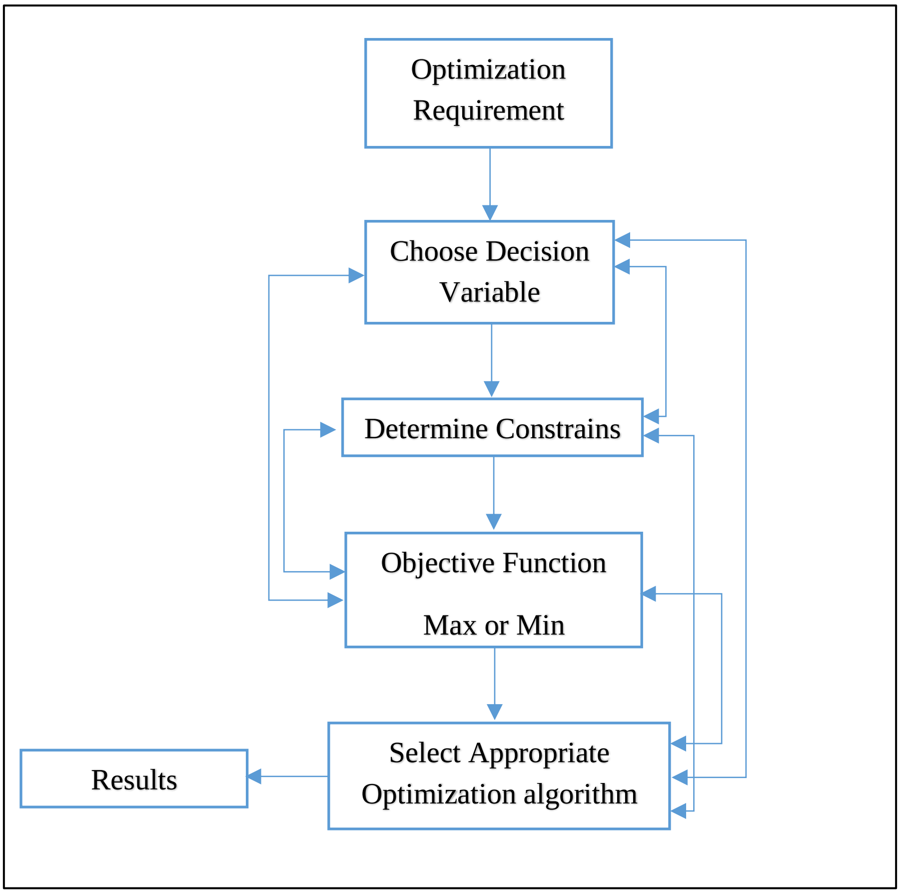


Figure 3-3: Mathematical Model Stages

## 3.6 Mathematical Model Solution

Based on the mathematical model type in terms of the mathematical form, the appropriate optimization technique is selected for model solution. Before using electronic calculators and computers, it was very difficult to solve complicated problems and mathematical models that contain a large number of information and equations. However, after the developments and advancements in software science, solving mathematical models has become an easy, highly, efficient and speedy process. There are numerous ready-made software programs that are produced to enable planners and decision making to solve complex problems and consequently take the appropriate decision. The person responsible has to select the appropriate method to solve the mathematical model based on the nature of the mathematical equations.

## 3.7 Sensitivity Analysis

It is not enough that a researcher arrives at the optimal solution of the model, but he should also obtain additional information on what might change in the model solution when the actual values of the decision variables in the model deviate from the suggested values. This additional information can be obtained through conducting sensitivity analysis. Changes calling for sensitivity analysis include:

1. Changes in available resources represented in the right-hand side of the constraints' equation.
2. Changes in constraint type.
3. Changes in variables coefficients, and target function.
4. Addition or removal of some constraints.

## 3.8 Model Validity

This refers to verification of that the proposed model has achieved the expected results; meaning answering the question: does the model give correct predictions to the behavior of the system under investigation?

The researcher should be convinced that the model outcomes do not contain any surprises; in other word that the solution is reasonable and achieves all the model condition and is likely to be applied. Verification can be accomplished through previous findings, if any. Further, verification can be carried out through investigation of the model's applicability by means of a questionnaire administered to specialists or beneficiaries.

# 4 Chapter Four: Model Formulation and Results Analysis

## 4.1 Introduction

The aim of this chapter is to construct a mathematical model and to solve it in compliance with the integer linear algorithmic programming technique which assists the company to ideally distribute sub-contractors to implement houses in accordance with area and method of implementation with the least possible cost and within time limit. Such as thing will also be the best investment for this type of contractors.

## 4.2 Problem of the Study

The problem of the study lies in the ideal distribution of house implementation assigned to six sub-contractors applying for the tender. It was based on the following areas: (A, B, C, D, E), method of implementation, first class (FD), second class (SD), to minimize the cost with estimated duration.

## 4.3 Study Data Collection

To construct a mathematical model that solves the problem of the study, the necessary information was collected from the company regarding tenders assigned for sub-contractors which include:

1. Every sub-contractor's estimated time needed to construct one house regarding area and degree of implementation.
2. Estimated cost for each house according to area and degree of implementation.
3. Time scheduled by the company for project implementation within 1400 days.
4. Budget assigned by the company for the project was $13 million. Tables (4-1) provide information from sub-contractors.

Table 4-1: Construction Cost and Time for Each sub-Contractor

| Sub-Contractor | No of Team | Type of House | Degree of House | Estimated Cost × $10^3$ \$/unit | Time Planned (days/unit) | Notes |
|---|---|---|---|---|---|---|
| 1st | One | A | F D | 73 | 39 | F D = First Degree<br>S D = Second Degree<br><br>1st and 2nd sub-contractor can be used two teams |
| | | | S D | 58 | 30 | |
| | | B | F D | 88 | 48 | |
| | | | S D | 70 | 37 | |
| | | C | F D | 97 | 55 | |
| | | | S D | 80 | 42 | |
| | | D | F D | 122 | 60 | |
| | | | S D | 96 | 45 | |
| | | E | F D | 152 | 70 | |
| | | | S D | 120 | 60 | |
| 2nd | One | A | F D | 75 | 38 | |
| | | | S D | 60 | 31 | |
| | | B | F D | 88 | 45 | |
| | | | S D | 70 | 33 | |
| | | C | F D | 98 | 60 | |
| | | | S D | 80 | 40 | |
| | | D | F D | 116 | 65 | |
| | | | S D | 96 | 41 | |
| | | E | F D | 150 | 75 | |
| | | | S D | 118 | 62 | |
| 3rd | One | A | F D | 70 | 42 | |
| | | | S D | 62 | 34 | |
| | | B | F D | 91 | 50 | |
| | | | S D | 73 | 40 | |
| | | C | F D | 102 | 60 | |
| | | | S D | 80 | 43 | |
| | | D | F D | 122 | 62 | |
| | | | S D | 99 | 48 | |
| | | E | F D | 150 | 72 | |
| | | | S D | 123 | 58 | |
| 4th | One | A | F D | 70 | 40 | Type of Houses<br>A=150 $m^2$<br>B=180 $m^2$<br>C=200 $m^2$<br>D=250 $m^2$<br>E=300 $m^2$ |
| | | | S D | 62 | 35 | |
| | | B | F D | 91 | 54 | |
| | | | S D | 73 | 40 | |
| | | C | F D | 102 | 60 | |
| | | | S D | 80 | 40 | |
| | | D | F D | 122 | 60 | |
| | | | S D | 99 | 44 | |
| | | E | F D | 150 | 72 | |
| | | | S D | 123 | 45 | |

Continued Table 4-1

| Sub-Contractor | No of Team | Type of House | Degree of House | Estimated Cost $\times 10^3$ \$/unit | Time Planned (days/unit) | Notes |
|---|---|---|---|---|---|---|
| 5th | One | A | F D | 75 | 45 | 1st sub-Contractor and 2nd sub-Contractor Can be Used Two Teams |
| | | | S D | 61 | 30 | |
| | | B | F D | 89 | 52 | |
| | | | S D | 71 | 40 | |
| | | C | F D | 96 | 56 | |
| | | | S D | 77 | 40 | |
| | | D | F D | 120 | 64 | |
| | | | S D | 95 | 40 | |
| | | E | F D | 153 | 77 | |
| | | | S D | 120 | 56 | |
| 6th | One | A | F D | 74 | 45 | |
| | | | S D | 58 | 35 | |
| | | B | F D | 92 | 48 | |
| | | | S D | 70 | 38 | |
| | | C | F D | 102 | 62 | |
| | | | S D | 83 | 43 | |
| | | D | F D | 127 | 63 | |
| | | | S D | 99 | 45 | |
| | | E | F D | 152 | 77 | |
| | | | S D | 122 | 56 | |

## 4.4 Model Formulation for Case Study (1)

Mathematical model was formulated in the research was ILP which comprised of many parts including.

### 4.4.1 Decision Variables

The variable for model was:

$H_{ijk}$: number of house type, construction degree k constructed by sub-contractor i.

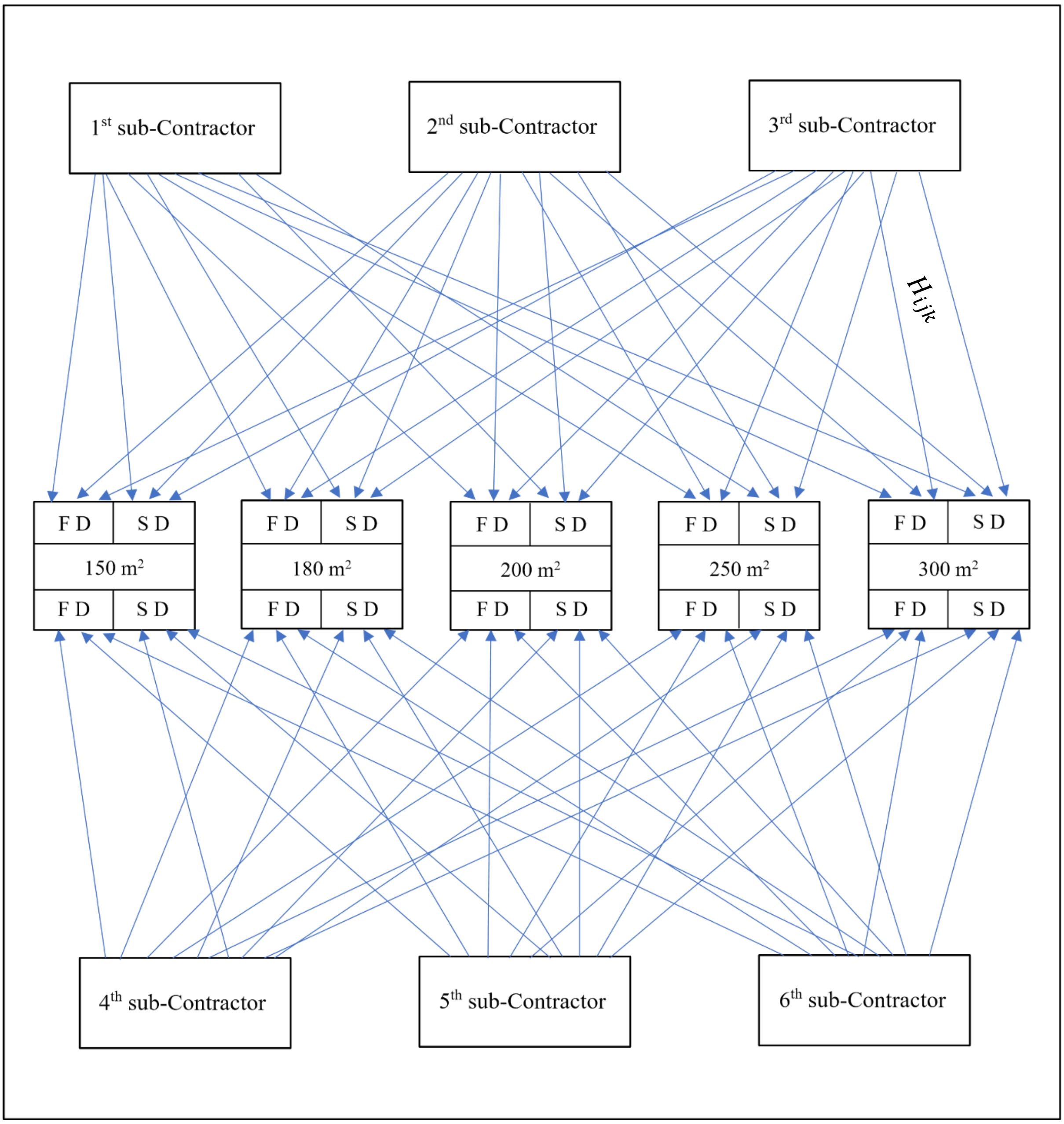

Figure 4-1: The Relationship of All Variables

### 4.4.2 Objective Function

The research objective function was minimized cost of project, defined as follow:

$$Min\ Z = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{D} C_{ijk} H_{ijk}$$

$c_{ijk}$ construction cost of sub-contractor I, house j and construction degree k in thousand dollars.

$i = 1,2, \dots, n \qquad n = 6$

$j = 1,2, \dots, m \qquad m = 5$

$k = 1,2$

**Expand to**

$Min\ Z = 73H_{111} + 58H_{112} + 88H_{121} + 70H_{122} + 97H_{131} + 81H_{132} + 122H_{141} + 96H_{142} + 152H_{151} + 120H_{152} + 75H_{211} + 60H_{212} + 88H_{221} + 70H_{222} + 98H_{231} + 80H_{232} + 116H_{241} + 96H_{242} + 150H_{251} + 118H_{252} + 70H_{311} + 62H_{312} + 91H_{321} + 73H_{322} + 102H_{331} + 80H_{332} + 122H_{341} + 99H_{342} + 150H_{351} + 123H_{352} + 70H_{411} + 60H_{412} + 90H_{421} + 72H_{422} + 100H_{431} + 80H_{432} + 125H_{441} + 100H_{442} + 150H_{451} + 120H_{452} + 75H_{511} + 61H_{512} + 89H_{521} + 71H_{522} + 96H_{531} + 77H_{532} + 120H_{541} + 95H_{542} + 153H_{551} + 120H_{552} + 74H_{611} + 58H_{612} + 92H_{621} + 70H_{622} + 102H_{631} + 83H_{632} + 127H_{641} + 99H_{642} + 152H_{651} + 122H_{652}$

### 4.4.3 Constraints

#### 4.4.3.1 Number of Houses

The total number of houses required (170), for each type shown in Table (4-2).

Table 4-2: Number of Each House Type

| **House Type** | A | B | C | D | E |
|---|---|---|---|---|---|
| **Must be equal** | 50 | 50 | 30 | 25 | 15 |

1. **House (A)**

$$\sum_{i=1}^{6}\sum_{k=1}^{2} H_{i1k} = 50$$

$$H_{111} + H_{112} + H_{211} + H_{212} + H_{311} + H_{312} + H_{411} + H_{412} + H_{511} + H_{512} + H_{611} + H_{612} = 50$$

2. **House (B)**

$$\sum_{i=1}^{6}\sum_{k=1}^{2} H_{i2k} = 50$$

$$H_{121} + H_{122} + H_{221} + H_{222} + H_{321} + H_{322} + H_{421} + H_{422} + H_{521} + H_{522} + H_{621} + H_{622} = 50$$

3. **House (C)**

$$\sum_{i=1}^{6}\sum_{k=1}^{2} H_{i3k} = 30$$

$$H_{131} + H_{132} + H_{231} + H_{232} + H_{331} + H_{332} + H_{431} + H_{432} + H_{531} + H_{532} + H_{631} + H_{632} = 30$$

4. **House (D)**

$$\sum_{i=1}^{6}\sum_{k=1}^{2} H_{i4k} = 25$$

$$H_{141} + H_{142} + H_{241} + H_{242} + H_{341} + H_{342} + H_{441} + H_{442} + H_{541} + H_{542} + H_{641} + H_{642} = 25$$

5. **House (E)**

$$\sum_{i=1}^{6}\sum_{k=1}^{2} H_{i5k} = 15$$

$$H_{151} + H_{152} + H_{251} + H_{252} + H_{351} + H_{352} + H_{451} + H_{452} + H_{551} + H_{552} + H_{651} + H_{652} = 15$$

#### 4.4.3.2 Project Budget

The budget of the project was limited; (B) and B equal (13 Million) $.

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k}^{D} C_{ijk}\,H_{ijk} \le B$$

**Expanded to**

$73H_{111} + 58H_{112} + 88H_{121} + 70H_{122} + 97H_{131} + 81H_{132} + 122H_{141} + 96H_{142} + 152H_{151} + 120H_{152} + 75H_{211} + 60H_{212} + 88H_{221} + 70H_{222} + 98H_{231} + 80H_{232} + 116H_{241} + 96H_{242} + 150H_{251} + 118H_{252} + 70H_{311} + 62H_{312} + 91H_{321} + 73H_{322} + 102H_{331} + 80H_{332} + 122H_{341} + 99H_{342} + 150H_{351} + 123H_{352} + 70H_{411} + 60H_{412} + 90H_{421} + 72H_{422} + 100H_{431} + 80H_{432} + 125H_{441} + 100H_{442} + 150H_{451} + 120H_{452} + 75H_{511} + 61H_{512} + 89H_{521} + 71H_{522} + 96H_{531} + 77H_{532} + 120H_{541} + 95H_{542} + 153H_{551} + 120H_{552} + 74H_{611} + 58H_{612} + 92H_{621} + 70H_{622} + 102H_{631} + 83H_{632} + 127H_{641} + 99H_{642} + 152H_{651} + 122H_{652} \le 13000$

#### 4.4.3.3 Project Duration

Duration for project less than or equal 1400 days.

$t_{ijk}$: duration estimating from sub-contractor $i$ per type $j$, construction degree $k$

1. **First sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{1jk}H_{1jk} \le 1400$$

$$38H_{111} + 30H_{112} + 48H_{121} + 37H_{122} + 55H_{131} + 42H_{132} + 60H_{141} + 45H_{142} + 70H_{151} + 60H_{152} \leq 1400$$

2. **Second sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{2jk}H_{2jk} \leq 1400$$

$$38H_{211} + 31H_{212} + 45H_{221} + 33H_{222} + 60H_{231} + 40H_{232} + 65H_{241} + 41H_{242} + 75H_{251} + 62H_{252} \leq 1400$$

3. **Third sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{3jk}H_{3jk} \leq 1400$$

$$42H_{311} + 34H_{312} + 50H_{321} + 40H_{322} + 60H_{331} + 43H_{332} + 62H_{341} + 48H_{342} + 72H_{351} + 58H_{352} \leq 1400$$

4. **Fourth sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{4jk}H_{4jk} \leq 1400$$

$$40H_{411} + 35H_{412} + 54H_{421} + 40H_{422} + 60H_{431} + 40H_{432} + 60H_{441} + 44H_{442} + 72H_{451} + 45H_{452} \leq 1400$$

5. **Fifth sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{5jk}H_{5jk} \leq 1400$$

$$45H_{511} + 30H_{512} + 52H_{521} + 40H_{522} + 56H_{531} + 40H_{532} + 64H_{541} + 40H_{542} + 77H_{551} + 56H_{552} \leq 1400$$

6. **Sixth sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{6jk}H_{6jk} \leq 1400$$

$$45H_{611} + 35H_{612} + 48H_{621} + 38H_{622} + 62H_{631} + 43H_{632} + 63H_{641} + 54H_{642} + 77H_{651} + 56H_{652} \leq 1400$$

$$H_{ijk} \geq 0 \; and \; integer$$

## 4.5 Solution Using MATLAB

MATLAB is a very well-known language for technical computing used by student, engineers, researchers and scientists in universities, and industries all everywhere throughout the world. This software is popular due to the fact it is powerful and easy to use. MATLAB can be used for mathematics computations, modeling and simulations, data analysis and processing, visualization and graphics, and algorithm development.

In this section, a systematic procedure for solving linear program problems is presented in details. The problem model equations represent a linear equations algorithm and this system can be solved by using the linear programming built in MATLAB software (Dantzig et al., 1955; Mehrotra, 1992; Zhang, 1995).

The function for the solution of the linear programming system is built in MATLAB program version (7.9). This function is named (Linprog), which solves a system of linear equation programming problem, of the following example syntax:

Finds the minimum of a problem specified by:

$$\min_{x} f^{T}x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq, \\ lb \leq x \leq ub. \end{cases}$$

```
x = linprog(f,A,b,Aeq,beq,lb,ub)
```

*f*, *x*, *b*, *beq*, *lb*, and *ub* are vectors, and *A* and *Aeq* are matrices.

In such case, the problem can be defined as:

Solves Min/Max $f * x$ such that $A * x \leq b$ or $A * xb$. And defines a set of lower and upper bounds on the design variables, $x$, so that the solution is always in the range $\mathrm{lb} \leq \mathrm{x} \leq \mathrm{ub}$. Includes equality constraints $\mathrm{Aeq} * \mathrm{x} = \mathrm{beq}$.

The governing equation of urban renewal programming as represented in equation was solved using the "**linprog**" built-in MATLAB function. Appendix A illustrates the MATLAB program using the present study to solve the problem system. It should be emphases that prior programming and solving the system, these equations must be converted to forms of MATLAB functions language. This process is very important and represented a key tool for solving the mathematical model in MATLAB.

## 4.6 Results Discussion of Case Study (1)

After importing available information of the mathematical model of case study (1) regarding goal function variable, coefficients of variables and available resources of each constants into ILP Software (Linprog), the outcomes achieved are presented in Table (4-3) which reveal that:

1. The minimal implementation cost was (12.907 M$) within the scheduled period set by the company, which was (1400) days, for project completion.
2. The study unveiled the ideal distribution to sub-contractors to implement houses with reference to type and degree of implementation.
3. The share of the second sub-contractor from those houses was the highest (3.088 M$), while the third sub-contractor was excluded because of his relatively high cost.

Table 4-3: Model Solution Case Study 1

<table>
<tr><th rowspan="3">Sub-Contractor</th><th colspan="10">No. of House $H_{ijk}$</th><th colspan="2" rowspan="2">Each sub-Contractor</th></tr>
<tr><th colspan="2">A</th><th colspan="2">B</th><th colspan="2">C</th><th colspan="2">D</th><th colspan="2">E</th></tr>
<tr><th>FD</th><th>SD</th><th>FD</th><th>SD</th><th>FD</th><th>SD</th><th>FD</th><th>SD</th><th>FD</th><th>SD</th><th>Cost M $</th><th>Duration (day)</th></tr>
<tr><td>1st</td><td>0</td><td>46</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>2.668</td><td>1380</td></tr>
<tr><td>2nd</td><td>0</td><td>0</td><td>0</td><td>15</td><td>0</td><td>0</td><td>0</td><td>20</td><td>0</td><td>1</td><td>3.088</td><td>1377</td></tr>
<tr><td>3rd</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>4th</td><td>0</td><td>1</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>14</td><td>1.812</td><td>705</td></tr>
<tr><td>5th</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>30</td><td>0</td><td>5</td><td>0</td><td>0</td><td>2.785</td><td>1400</td></tr>
<tr><td>6th</td><td>0</td><td>3</td><td>0</td><td>34</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>2.554</td><td>1397</td></tr>
<tr><td>Total of House</td><td colspan="2">50</td><td colspan="2">50</td><td colspan="2">30</td><td colspan="2">25</td><td colspan="2">15</td><td colspan="2">-</td></tr>
<tr><td>Project Cost M $</td><td colspan="12">12.907</td></tr>
<tr><td>Duration (day)</td><td colspan="12">1400</td></tr>
</table>

## 4.7 Sensitivity Model Analysis

The major objective of the first case study was to come to an ideal solution for the integer linear programming model. Due to the need to discuss all probabilities that might influence the value of goal function, it was mandatory to investigate such factors and their impacts. Sensitivity analysis is presented as follows:

1. Changing some restrictions and constraint forms (=, ≥, ≤);
2. Changing available resource for some constraint;
3. Excluding or adding new variables;
4. Adding new restrictions and constraint.

They were all executed as new study cases (2, 3, 4, 5, 6, 7)

## 4.8 Case Study (2)

This case study included:

1. Changing the constraints of the number of houses to be implemented from each type (A, B, C, D, E) as presented in table (4-4).
2. Adding new constraints that the total number of houses for each type with regard to area and implementation method equals (170). Not adding these constraints will lead to opting for a fewer number of houses, so the result will eventually be less than the required number.

The mathematical model of case study (2) as follow:

### 4.8.1 Objective Function

$$Min\ Z = \sum_{i=1}^{6} \sum_{j=1}^{5} \sum_{k=1}^{2} C_{ijk}\, K_{ijk}$$

$$Min\ Z = 73H_{111} + 58H_{112} + 88H_{121} + 70H_{122} + 97H_{131} + 81H_{132} + 122H_{141} + 96H_{142} + 152H_{151} + 120H_{152} + 75H_{211} + 60H_{212} + 88H_{221} + 70H_{222} + 98H_{231} + 80H_{232} + 116H_{241} + 96H_{242} + 150H_{251} + 118H_{252} + 70H_{311} + 62H_{312} + 91H_{321} + 73H_{322} + 102H_{331} + 80H_{332} + 122H_{341} + 99H_{342} + 150H_{351} + 123H_{352} + 70H_{411} + 60H_{412} + 90H_{421} + 72H_{422} + 100H_{431} + 80H_{432} + 125H_{441} + 100H_{442} + 150H_{451} + 120H_{452} + 75H_{511} + 61H_{512} + 89H_{521} + 71H_{522} + 96H_{531} + 77H_{532} + 120H_{541} + 95H_{542} + 153H_{551} + 120H_{552} + 74H_{611} + 58H_{612} + 92H_{621} + 70H_{622} + 102H_{631} + 83H_{632} + 127H_{641} + 99H_{642} + 152H_{651} + 122H_{652}$$

## 4.8.2 Constraints

### 4.8.2.1 Number of Houses

The total number of houses required (170), for each type according Table (4-4)

Table 4-4: Number of Houses

| House Type | A | B | C | D | E |
|---|---|---|---|---|---|
| Less Than or Equal | 60 | 60 | 35 | 35 | 25 |
| More Than or Equal | 40 | 40 | 25 | 20 | 10 |

1. **House A**

$$\sum_{i=1}^{6} \sum_{k=1}^{2} H_{i1k} \leq 60$$

$$\sum_{i=1}^{6} \sum_{k=1}^{2} H_{i1k} \geq 40$$

$$H_{111} + H_{112} + H_{211} + H_{212} + H_{311} + H_{312} + H_{411} + H_{412} + H_{511} + H_{512} + H_{611} + H_{612} \leq 60$$

$$H_{111} + H_{112} + H_{211} + H_{212} + H_{311} + H_{312} + H_{411} + H_{412} + H_{511} + H_{512} + H_{611} + H_{612} \geq 40$$

2. **House B**

$$\sum_{i=1}^{6} \sum_{k=1}^{2} H_{i2k} \leq 60$$

$$\sum_{i=1}^{6} \sum_{k=1}^{2} H_{i2k} \geq 40$$

$$H_{121} + H_{122} + H_{221} + H_{222} + H_{321} + H_{322} + H_{421} + H_{422} + H_{521} + H_{522} + H_{621} + H_{622} \leq 60$$

$$H_{121} + H_{122} + H_{221} + H_{222} + H_{321} + H_{322} + H_{421} + H_{422} + H_{521} + H_{522} + H_{621} + H_{622} \geq 40$$

3. **House C**

$$\sum_{i=1}^{6} \sum_{k=1}^{2} H_{i3k} \leq 35$$

$$\sum_{i=1}^{6} \sum_{k=1}^{2} H_{i3k} \geq 25$$

$$H_{131} + H_{132} + H_{231} + H_{232} + H_{331} + H_{332} + H_{431} + H_{432} + H_{531} + H_{532} + H_{631} + H_{632} \leq 35$$

$$H_{131} + H_{132} + H_{231} + H_{232} + H_{331} + H_{332} + H_{431} + H_{432} + H_{531} + H_{532} + H_{631} + H_{632} \geq 25$$

4. **House D**

$$\sum_{i=1}^{6}\sum_{k=1}^{2} H_{i4k} \leq 35$$

$$\sum_{i=1}^{6}\sum_{k=1}^{2} H_{i4k} \geq 20$$

$$H_{141} + H_{142} + H_{241} + H_{242} + H_{341} + H_{342} + H_{441} + H_{442} + H_{541} + H_{542} + H_{641} + H_{642} \geq 35$$

$$H_{141} + H_{142} + H_{241} + H_{242} + H_{341} + H_{342} + H_{441} + H_{442} + H_{541} + H_{542} + H_{641} + H_{642} \geq 20$$

5. **House E**

$$\sum_{i=1}^{6}\sum_{k=1}^{2} H_{i5k} \leq 25$$

$$\sum_{i=1}^{6}\sum_{k=1}^{2} H_{i5k} \geq 10$$

$$H_{151} + H_{152} + H_{251} + H_{252} + H_{351} + H_{352} + H_{451} + H_{452} + H_{551} + H_{552} + H_{651} + H_{652} \leq 25$$

$$H_{151} + H_{152} + H_{251} + H_{252} + H_{351} + H_{352} + H_{451} + H_{452} + H_{551} + H_{552} + H_{651} + H_{652} \geq 10$$

#### 4.8.2.2 Project Budget

The budget of project was limited ;( B), and B equal (13Million) $

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k}^{D} C_{ijk}\, H_{ijk} \leq B$$

**Expanded to**

$$73H_{111} + 58H_{112} + 88H_{121} + 70H_{122} + 97H_{131} + 81H_{132} + 122H_{141} + 96H_{142} + 152H_{151} + 120H_{152} + 75H_{211} + 60H_{212} + 88H_{221} + 70H_{222} + 98H_{231} + 80H_{232} + 116H_{241} + 96H_{242} + 150H_{251} + 118H_{252} + 70H_{311} + 62H_{312} + 91H_{321} + 73H_{322} + 102H_{331} + 80H_{332} + 122H_{341} + 99H_{342} + 150H_{351} + 123H_{352} + 70H_{411} + 60H_{412} + 90H_{421} + 72H_{422} + 100H_{431} + 80H_{432} + 125H_{441} + 100H_{442} + 150H_{451} + 120H_{452} + 75H_{511} + 61H_{512} + 89H_{521} + 71H_{522} + 96H_{531} + 77H_{532} + 120H_{541} + 95H_{542} + 153H_{551} + 120H_{552} + 74H_{611} + 58H_{612} + 92H_{621} + 70H_{622} + 102H_{631} + 83H_{632} + 127H_{641} + 99H_{642} + 152H_{651} + 122H_{652} \leq 13000$$

#### 4.8.2.3 Project Duration for Case Study (2)

Duration for project less than or equal 1400 days.

**1. First sub-Contractor**

$$\sum_{j=1}^{5} \sum_{k=1}^{2} t_{1jk} H_{1jk} \leq 1400$$

$$38H_{111} + 30H_{112} + 48H_{121} + 37H_{122} + 55H_{131} + 42H_{132} + 60H_{141} + 45H_{142} + 70H_{151} + 60H_{152} \leq 1400$$

**2. Second sub-Contractor**

$$\sum_{j=1}^{5} \sum_{k=1}^{2} t_{2jk} H_{2jk} \leq 1400$$

$$38H_{211} + 31H_{212} + 45H_{221} + 33H_{222} + 60H_{231} + 40H_{232} + 65H_{241} + 41H_{242} + 75H_{251} + 62H_{252} \leq 1400$$

3. **Third sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{3jk}H_{3jk} \leq 1400$$

$$42H_{311} + 34H_{312} + 50H_{321} + 40H_{322} + 60H_{331} + 43H_{332} + 62H_{341} + 48H_{342} + 72H_{351} + 58H_{352} \leq 1400$$

4. **Fourth sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{4jk}H_{4jk} \leq 1400$$

$$40H_{411} + 35H_{412} + 54H_{421} + 40H_{422} + 60H_{431} + 40H_{432} + 60H_{441} + 44H_{442} + 72H_{451} + 45H_{452} \leq 1400$$

5. **Fifth sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{5jk}H_{5jk} \leq 1400$$

$$45H_{511} + 30H_{512} + 52H_{521} + 40H_{522} + 56H_{531} + 40H_{532} + 64H_{541} + 40H_{542} + 77H_{551} + 56H_{552} \leq 1400$$

6. **Sixth sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{6jk}H_{6jk} \leq 1400$$

$$45H_{611} + 35H_{612} + 48H_{621} + 38H_{622} + 62H_{631} + 43H_{632} + 63H_{641} + 54H_{642} + 77H_{651} + 56H_{652} \leq 1400$$

#### 4.8.2.4 Total Number of Houses Equal (170)

$$\sum_{i=1}^{6}\sum_{j=1}^{5}\sum_{k=1}^{2} H_{ijk} = 170$$

$$H_{111} + H_{112} + H_{121} + H_{122} + H_{131} + H_{132} + H_{141} + H_{142} + H_{151} + H_{152} + H_{211} + H_{212} + H_{221} + H_{222} + H_{231} + H_{232} + H_{241} + H_{242} + H_{251} + H_{252} + H_{311} + H_{312} + H_{321} + H_{322} + H_{331} + H_{332} + H_{341} + H_{342} + H_{351} + H_{352} + H_{411} + H_{412} + H_{421} + H_{422} + H_{431} + H_{432} + H_{441} + H_{442} + H_{451} + H_{452} + H_{511} + H_{512} + H_{521} + H_{522} + H_{531} + H_{532} + H_{541} + H_{542} + H_{551} + H_{552} + H_{611} + H_{612} + H_{621} + H_{622} + H_{631} + H_{632} + H_{641} + H_{642} + H_{651} + H_{652} = 170$$

$$H_{ijk} \geq 0 \; and \; integer$$

## 4.9 Results Discussion of Case Study (2)

Table (4-5) presents the results achieved from this case study of the mathematical model as follows:

1. The total cost for project implementation was (12.371) M$ which is less than that of the first case study (0.536) M$. This indicates an improvement of the model solution within the company's plan of (1400) work days of the project.
2. In the model distribution of sub-contractors for constructing houses, the biggest share was the second sub-contractor with a total cost of (2.940) M$; the third sub-contractor was excluded.

3. The number of houses distributed for implementation differed from the first case study, with regard to type.

Table 4-5: Model Solution Case Study 2

| Sub-Contractor | No. of House $H_{ijk}$ | | | | | | | | | | Each sub-Contractor | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | | B | | C | | D | | E | | | |
| | FD | SD | FD | SD | FD | SD | FD | SD | FD | SD | Cost M $ | Duration (day) |
| $1^{st}$ | 0 | 33 | 0 | 0 | 0 | 0 | 0 | 9 | 0 | 0 | 2.778 | 1395 |
| $2^{nd}$ | 0 | 0 | 0 | 42 | 0 | 0 | 0 | 0 | 0 | 0 | 2.94 | 1386 |
| $3^{rd}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $4^{th}$ | 0 | 2 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 10 | 1.400 | 700 |
| $5^{th}$ | 0 | 0 | 0 | 0 | 0 | 24 | 0 | 11 | 0 | 0 | 2.893 | 1400 |
| $6^{th}$ | 0 | 25 | 0 | 13 | 0 | 0 | 0 | 0 | 0 | 0 | 2.360 | 1369 |
| Total of House | 60 | | 55 | | 25 | | 20 | | 10 | | - | |
| Project Cost M $ | 12.371 | | | | | | | | | | | |
| Duration (day) | 1400 | | | | | | | | | | | |

## 4.10 Case Study (3)

Because the total cost of case study (2) was less than that of case study one, a model for case study (3) was designed and based on case study (2). Available resources pertaining project time period were changed to become (1200) days instead of (1400).

The mathematical model of case study (3) as follow:

### 4.10.1 Objective Function

$$Min\ Z = \sum_{i=1}^{6} \sum_{j=1}^{5} \sum_{k=1}^{2} C_{ijk}\, K_{ijk}$$

$$Min\ Z = 73H_{111} + 58H_{112} + 88H_{121} + 70H_{122} + 97H_{131} + 81H_{132} +$$

$$122H_{141} + 96H_{142} + 152H_{151} + 120H_{152} + 75H_{211} + 60H_{212} + 88H_{221} + 70H_{222} +$$

$98H_{231} + 80H_{232} + 116H_{241} + 96H_{242} + 150H_{251} + 118H_{252} + 70H_{311} + 62H_{312} +$

$91H_{321} + 73H_{322} + 102H_{331} + 80H_{332} + 122H_{341} + 99H_{342} + 150H_{351} +$

$123H_{352} + 70H_{411} + 60H_{412} + 90H_{421} + 72H_{422} + 100H_{431} + 80H_{432} +$

$125H_{441} + 100H_{442} + 150H_{451} + 120H_{452} + 75H_{511} + 61H_{512} + 89H_{521} +$

$71H_{522} + 96H_{531} + 77H_{532} + 120H_{541} + 95H_{542} + 153H_{551} + 120H_{552} +$

$74H_{611} + 58H_{612} + 92H_{621} + 70H_{622} + 102H_{631} + 83H_{632} + 127H_{641} + 99H_{642} +$

$152H_{651} + 122H_{652}$

### 4.10.2 Constraints

#### 4.10.2.1 Number of Houses

The total number of houses required (170), for each type according Table (4-6)

Table 4-6: Number of Houses

| House Type | A | B | C | D | E |
|---|---|---|---|---|---|
| Less Than or Equal | 60 | 60 | 35 | 35 | 25 |
| More Than or Equal | 40 | 40 | 25 | 20 | 10 |

1. **House A**

$$\sum_{i=1}^{6} \sum_{k=1}^{2} H_{i1k} \leq 60$$

$$\sum_{i=1}^{6} \sum_{k=1}^{2} H_{i1k} \geq 40$$

$$H_{111} + H_{112} + H_{211} + H_{212} + H_{311} + H_{312} + H_{411} + H_{412} + H_{511} + H_{512} + H_{611} + H_{612} \leq 60$$

$$H_{111} + H_{112} + H_{211} + H_{212} + H_{311} + H_{312} + H_{411} + H_{412} + H_{511} + H_{512} + H_{611} + H_{612} \geq 40$$

2. **House B**

$$\sum_{i=1}^{6} \sum_{k=1}^{2} H_{i2k} \leq 60$$

$$\sum_{i=1}^{6} \sum_{k=1}^{2} H_{i2k} \geq 40$$

$$H_{121} + H_{122} + H_{221} + H_{222} + H_{321} + H_{322} + H_{421} + H_{422} + H_{521} + H_{522} + H_{621} + H_{622} \leq 60$$

$$H_{121} + H_{122} + H_{221} + H_{222} + H_{321} + H_{322} + H_{421} + H_{422} + H_{521} + H_{522} + H_{621} + H_{622} \geq 40$$

3. **House C**

$$\sum_{i=1}^{6} \sum_{k=1}^{2} H_{i3k} \leq 35$$

$$\sum_{i=1}^{6} \sum_{k=1}^{2} H_{i3k} \geq 25$$

$$H_{131} + H_{132} + H_{231} + H_{232} + H_{331} + H_{332} + H_{431} + H_{432} + H_{531} + H_{532} + H_{631} + H_{632} \leq 35$$

$$H_{131} + H_{132} + H_{231} + H_{232} + H_{331} + H_{332} + H_{431} + H_{432} + H_{531} + H_{532} + H_{631} + H_{632} \geq 25$$

4. **House D**

$$\sum_{i=1}^{6}\sum_{k=1}^{2} H_{i4k} \leq 35$$

$$\sum_{i=1}^{6}\sum_{k=1}^{2} H_{i4k} \geq 20$$

$$H_{141} + H_{142} + H_{241} + H_{242} + H_{341} + H_{342} + H_{441} + H_{442} + H_{541} + H_{542} + H_{641} + H_{642} \geq 35$$

$$H_{141} + H_{142} + H_{241} + H_{242} + H_{341} + H_{342} + H_{441} + H_{442} + H_{541} + H_{542} + H_{641} + H_{642} \geq 20$$

5. **House E**

$$\sum_{i=1}^{6}\sum_{k=1}^{2} H_{i5k} \leq 25$$

$$\sum_{i=1}^{6}\sum_{k=1}^{2} H_{i5k} \geq 10$$

$$H_{151} + H_{152} + H_{251} + H_{252} + H_{351} + H_{352} + H_{451} + H_{452} + H_{551} + H_{552} + H_{651} + H_{652} \leq 25$$

$$H_{151} + H_{152} + H_{251} + H_{252} + H_{351} + H_{352} + H_{451} + H_{452} + H_{551} + H_{552} + H_{651} + H_{652} \geq 10$$

#### 4.10.2.2 Project Budget

The budget of project was limited ;( B), and B equal (13Million) $

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k}^{D} C_{ijk}\, H_{ijk} \leq B$$

**Expanded to**

$$73H_{111} + 58H_{112} + 88H_{121} + 70H_{122} + 97H_{131} + 81H_{132} + 122H_{141} + 96H_{142} + 152H_{151} + 120H_{152} + 75H_{211} + 60H_{212} + 88H_{221} + 70H_{222} + 98H_{231} + 80H_{232} + 116H_{241} + 96H_{242} + 150H_{251} + 118H_{252} + 70H_{311} + 62H_{312} + 91H_{321} + 73H_{322} + 102H_{331} + 80H_{332} + 122H_{341} + 99H_{342} + 150H_{351} + 123H_{352} + 70H_{411} + 60H_{412} + 90H_{421} + 72H_{422} + 100H_{431} + 80H_{432} + 125H_{441} + 100H_{442} + 150H_{451} + 120H_{452} + 75H_{511} + 61H_{512} + 89H_{521} + 71H_{522} + 96H_{531} + 77H_{532} + 120H_{541} + 95H_{542} + 153H_{551} + 120H_{552} + 74H_{611} + 58H_{612} + 92H_{621} + 70H_{622} + 102H_{631} + 83H_{632} + 127H_{641} + 99H_{642} + 152H_{651} + 122H_{652} \leq 13000$$

#### 4.10.2.3 Project Duration for Case Study (3)

Duration for project less than or equal 1200 days.

1. **First sub-Contractor**

$$\sum_{j=1}^{5} \sum_{k=1}^{2} t_{1jk} H_{1jk} \leq 1200$$

$$38H_{111} + 30H_{112} + 48H_{121} + 37H_{122} + 55H_{131} + 42H_{132} + 60H_{141} + 45H_{142} + 70H_{151} + 60H_{152} \leq 1200$$

2. **Second sub-Contractor**

$$\sum_{j=1}^{5} \sum_{k=1}^{2} t_{2jk} H_{2jk} \leq 1200$$

$$38H_{211} + 31H_{212} + 45H_{221} + 33H_{222} + 60H_{231} + 40H_{232} + 65H_{241} + 41H_{242} + 75H_{251} + 62H_{252} \leq 1200$$

3. **Third sub-Contractor**

$$\sum_{j=1}^{5} \sum_{k=1}^{2} t_{3jk} H_{3jk} \leq 1200$$

$$42H_{311} + 34H_{312} + 50H_{321} + 40H_{322} + 60H_{331} + 43H_{332} + 62H_{341} + 48H_{342} + 72H_{351} + 58H_{352} \leq 1200$$

4. **Fourth sub-Contractor**

$$\sum_{j=1}^{5} \sum_{k=1}^{2} t_{4jk} H_{4jk} \leq 1200$$

$$40H_{411} + 35H_{412} + 54H_{421} + 40H_{422} + 60H_{431} + 40H_{432} + 60H_{441} + 44H_{442} + 72H_{451} + 45H_{452} \leq 1200$$

5. **Fifth sub-Contractor**

$$\sum_{j=1}^{5} \sum_{k=1}^{2} t_{5jk} H_{5jk} \leq 1200$$

$$45H_{511} + 30H_{512} + 52H_{521} + 40H_{522} + 56H_{531} + 40H_{532} + 64H_{541} + 40H_{542} + 77H_{551} + 56H_{552} \leq 1200$$

6. **Sixth sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{6jk}H_{6jk} \leq 1200$$

$$45H_{611} + 35H_{612} + 48H_{621} + 38H_{622} + 62H_{631} + 43H_{632} + 63H_{641} + 54H_{642} + 77H_{651} + 56H_{652} \leq 1200$$

#### 4.10.2.4 Total Number of Houses Equal (170)

$$\sum_{i=1}^{6}\sum_{j=1}^{5}\sum_{k=1}^{2} H_{ijk} = 170$$

$$H_{111} + H_{112} + H_{121} + H_{122} + H_{131} + H_{132} + H_{141} + H_{142} + H_{151} + H_{152} + H_{211} + H_{212} + H_{221} + H_{222} + H_{231} + H_{232} + H_{241} + H_{242} + H_{251} + H_{252} + H_{311} + H_{312} + H_{321} + H_{322} + H_{331} + H_{332} + H_{341} + H_{342} + H_{351} + H_{352} + H_{411} + H_{412} + H_{421} + H_{422} + H_{431} + H_{432} + H_{441} + H_{442} + H_{451} + H_{452} + H_{511} + H_{512} + H_{521} + H_{522} + H_{531} + H_{532} + H_{541} + H_{542} + H_{551} + H_{552} + H_{611} + H_{612} + H_{621} + H_{622} + H_{631} + H_{632} + H_{641} + H_{642} + H_{651} + H_{652} = 170$$

$$H_{ijk} \geq 0 \; and \; integer$$

### 4.10.3 Results Discussion Case Study (3)

Table (4-7) illustrates outputs of the mathematical model of case study (3) where the results showed a new better distribution of sub-contractors for implementation of residential houses; the fifth sub-contractor share was the biggest and none of the sub-contractors was excluded.

The total cost was (12.412) M$, higher than its counterpart of the second case study, but the period needed for project completion was (1200) days, less than the two previously studied cases. This allows company owner to choose the best from the two and eventually optimum for the least costly or timely shorter.

Table 4-7: Model Solution Case Study 3

<table>
<tr><th rowspan="3">Sub-Contractor</th><th colspan="10">No. of House $H_{ijk}$</th><th colspan="2">Each sub-Contractor</th></tr>
<tr><th colspan="2">A</th><th colspan="2">B</th><th colspan="2">C</th><th colspan="2">D</th><th colspan="2">E</th><th rowspan="2">Cost M $</th><th rowspan="2">Duration (day)</th></tr>
<tr><th>FD</th><th>SD</th><th>FD</th><th>SD</th><th>FD</th><th>SD</th><th>FD</th><th>SD</th><th>FD</th><th>SD</th></tr>
<tr><td>1st</td><td>0</td><td>40</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>9</td><td>0</td><td>0</td><td>2.320</td><td>1200</td></tr>
<tr><td>2nd</td><td>0</td><td>0</td><td>0</td><td>36</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>2.520</td><td>1188</td></tr>
<tr><td>3rd</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>11</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0.88</td><td>473</td></tr>
<tr><td>4th</td><td>0</td><td>0</td><td>0</td><td>6</td><td>0</td><td>4</td><td>0</td><td>0</td><td>0</td><td>10</td><td>1.952</td><td>950</td></tr>
<tr><td>5th</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>10</td><td>0</td><td>20</td><td>0</td><td>0</td><td>2.670</td><td>1200</td></tr>
<tr><td>6th</td><td>0</td><td>20</td><td>0</td><td>13</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>2.070</td><td>1194</td></tr>
<tr><td>Total of House</td><td colspan="2">60</td><td colspan="2">55</td><td colspan="2">25</td><td colspan="2">20</td><td colspan="2">10</td><td colspan="2">-</td></tr>
<tr><td>Project Cost M $</td><td colspan="12">12.412</td></tr>
<tr><td>Duration (day)</td><td colspan="12">1200</td></tr>
</table>

## 4.11 Case Study (4)

Results of case studies (1) and (2) unveiled that the third sub-contractor was not given any work. A few numbers of houses were assigned to the fourth sub-contractor. Because the sub-contractors (1 and 2) were able to employ two groups and based on the information in table (1-4), it was concluded that:

1. Decision variable for third and fourth contractors was excluded.
2. The period needed for unit completion was eventually reduced to half for the sub-contractors (1 and 2).
3. The constraints of number of houses with regard to type and equal area was retained, the mathematical model as follow:

### 4.11.1 Objective Function

$$Min\ Z = \sum_{i=1}^{4} \sum_{j=1}^{5} \sum_{k=1}^{2} C_{ijk}\, K_{ijk}$$

Where

$i = 1, 2, \ldots, 4$

$j = 1, 2, \ldots, 5$

$k = 1, 2$

$$Min\ Z = 73H_{111} + 58H_{112} + 88H_{121} + 70H_{122} + 97H_{131} + 81H_{132} + 122H_{141} + 96H_{142} + 152H_{151} + 120H_{152} + 75H_{211} + 60H_{212} + 88H_{221} + 70H_{222} + 98H_{231} + 80H_{232} + 116H_{241} + 96H_{242} + 150H_{251} + 118H_{252} + 75H_{511} + 61H_{512} + 89H_{521} + 71H_{522} + 96H_{531} + 77H_{532} + 120H_{541} + 95H_{542} + 153H_{551} + 120H_{552} + 74H_{611} + 58H_{612} + 92H_{621} + 70H_{622} + 102H_{631} + 83H_{632} + 127H_{641} + 99H_{642} + 152H_{651} + 122H_{652}$$

### 4.11.2 Constraints

#### 4.11.2.1 Number of Houses

The number of houses required (170), for each type according to Table (4-8).

Table 4-8: Number of Houses

| House Type | A | B | C | D | E |
|---|---|---|---|---|---|
| must by equal | 50 | 50 | 30 | 25 | 15 |

1. **House A**

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i1k} = 50$$

$$H_{111} + H_{112} + H_{211} + H_{212} + H_{511} + H_{512} + H_{611} + H_{612} = 50$$

**2. House B**

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i2k} = 50$$

$$H_{121} + H_{122} + H_{221} + H_{222} + H_{521} + H_{522} + H_{621} + H_{622} = 50$$

**3. House C**

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i3k} = 30$$

$$H_{131} + H_{132} + H_{231} + H_{232} + H_{531} + H_{532} + H_{631} + H_{632} = 30$$

**4. House D**

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i4k} = 25$$

$$H_{141} + H_{142} + H_{241} + H_{242} + H_{541} + H_{542} + H_{641} + H_{642} = 25$$

**5. House E**

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i5k} = 15$$

$$H_{151} + H_{152} + H_{251} + H_{252} + H_{551} + H_{552} + H_{651} + H_{652} = 15$$

#### 4.11.2.2 Budget Limit

$$\sum_{i=1}^{4}\sum_{j=1}^{5}\sum_{k}^{2} C_{ijk}\, H_{ijk} \leq B$$

**Expanded to**

$$73H_{111} + 58H_{112} + 88H_{121} + 70H_{122} + 97H_{131} + 81H_{132} + 122H_{141}$$
$$+ 96H_{142} + 152H_{151} + 120H_{152} + 75H_{211} + 60H_{212} + 88H_{221}$$
$$+ 70H_{222} + 98H_{231} + 80H_{232} + 116H_{241} + 96H_{242} + 150H_{251}$$
$$+ 118H_{252} + 75H_{511} + 61H_{512} + 89H_{521} + 71H_{522} + 96H_{531}$$
$$+ 77H_{532} + 120H_{541} + 95H_{542} + 153H_{551} + 120H_{552} + 74H_{611}$$
$$+ 58H_{612} + 92H_{621} + 70H_{622} + 102H_{631} + 83H_{632} + 127H_{641}$$
$$+ 99H_{642}152H_{651} + 122H_{652} \leq 13000$$

#### 4.11.2.3 Project Duration

1. **First Sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{1jk}H_{1jk} \leq 1400$$

$$19.5H_{111} + 15H_{112} + 24H_{121} + 18.5H_{122} + 27.5H_{131} + 21H_{132} + 30H_{141}$$
$$+ 22.5H_{142} + 35H_{151} + 30H_{152} \leq 1400$$

2. **Second Sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{2jk}H_{2jk} \leq 1400$$

$$19H_{211} + 15.5H_{212} + 22.5H_{221} + 16.5H_{222} + 30H_{231} + 20H_{232} + 32.5H_{241} + 20.5H_{242} + 37.5H_{251} + 31H_{252} \le 1400$$

**3. Fifth Sub-contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{5jk}H_{5jk} \le 1400$$

$$45H_{511} + 30H_{512} + 52H_{521} + 40H_{522} + 56H_{531} + 40H_{532} + 64H_{541} + 40H_{542} + 77H_{551} + 56H_{552} \le 1400$$

**4. Sixth Sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{6jk}H_{6jk} \le 1400$$

$$45H_{611} + 35H_{612} + 48H_{621} + 38H_{622} + 62H_{631} + 43H_{632} + 63H_{641} + 54H_{642} + 77H_{651} + 56H_{652} \le 1400$$

$$H_{ijk} \ge 0 \; and \; integer$$

## 4.12 Results Discussion of Case Study (4)

Table (4-9) illustrates outcomes of mathematical model of case study (4). The biggest share was that of the second sub-contractor for a total cost. of (4.410) and none of the sub-contractors was excluded. The total cost for project completion was (12.875), less than study case (1) and the time period needed for project completion was (1400) days.

Table 4-9: Model Solution Case Study 4

| Sub-Contractor | No. of House $H_{ijk}$ | | | | | | | | | | Each sub-Contractor | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | | B | | C | | D | | E | | Cost M $ | Duration (day) |
| | FD | SD | FD | SD | FD | SD | FD | SD | FD | SD | | |
| 1st | 0 | 32 | 0 | 14 | 0 | 0 | 0 | 10 | 0 | 0 | 3.796 | 964 |
| 2nd | 0 | 0 | 0 | 24 | 0 | 0 | 0 | 10 | 0 | 15 | 4.410 | 1066 |
| 5th | 0 | 0 | 0 | 0 | 0 | 30 | 0 | 5 | 0 | 0 | 2.785 | 1400 |
| 6th | 0 | 18 | 0 | 12 | 0 | 0 | 0 | 0 | 0 | 0 | 1.884 | 1086 |
| Total of House | 50 | | 50 | | 30 | | 25 | | 15 | | - | |
| Project Cost M $ | 12.875 | | | | | | | | | | | |
| Duration (day) | 1400 | | | | | | | | | | | |

## 4.13 Case Study (5)

Based on case study (4), a mathematical model was designed in which constraints of number of houses for each were changed being less than or equal, and more than or equal, in addition to required time period for project completion (1400) days, adding a determinant for the total number of houses equal to (170).

The mathematical model as follow:

### 4.13.1 Objective Function

$$Min\ Z = \sum_{i=1}^{4} \sum_{j=1}^{5} \sum_{k=1}^{2} C_{ijk}\, K_{ijk}$$

**Expanded to**

$$Min\ Z = 73H_{111} + 58H_{112} + 88H_{121} + 70H_{122} + 97H_{131} + 81H_{132} + 122H_{141} + 96H_{142} + 152H_{151} + 120H_{152} + 75H_{211} + 60H_{212} + 88H_{221} + 70H_{222} + 98H_{231} + 80H_{232} + 116H_{241} + 96H_{242} + 150H_{251} + 118H_{252} + 75H_{511} + 61H_{512} + 89H_{521} + 71H_{522} + 96H_{531} + 77H_{532} + 120H_{541} + 95H_{542} + 153H_{551} + 120H_{552} + 74H_{611} + 58H_{612} + 92H_{621} + 70H_{622} + 102H_{631} + 83H_{632} + 127H_{641} + 99H_{642} + 152H_{651} + 122H_{652}$$

### 4.13.2 Constraints

#### 4.13.2.1 Number of Houses

The total number of houses required (170), for each type according Table (4-10).

Table 4-10: Number of Houses

| House Type | A | B | C | D | E |
|---|---|---|---|---|---|
| Less Than or Equal | 60 | 60 | 35 | 35 | 25 |
| More Than or Equal | 40 | 40 | 25 | 20 | 10 |

1. **House A**

$$\sum_{i=1}^{4}\sum_{k=1}^{2} H_{i1k} \geq 40$$

$$H_{111} + H_{112} + H_{211} + H_{212} + H_{511} + H_{512} + H_{611} + H_{612} \geq 40$$

$$\sum_{i=1}^{4}\sum_{k=1}^{2} H_{i1k} \leq 60$$

$$H_{111} + H_{112} + H_{211} + H_{212} + H_{511} + H_{512} + H_{611} + H_{612} \leq 60$$

2. **House B**

$$\sum_{i=1}^{4}\sum_{k=1}^{2} H_{i2k} \geq 40$$

$$H_{121} + H_{122} + H_{221} + H_{222} + H_{521} + H_{522} + H_{621} + H_{622} \geq 40$$

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i2k} \leq 60$$

$$H_{121} + H_{122} + H_{221} + H_{222} + H_{521} + H_{522} + H_{621} + H_{622} \leq 60$$

**3. House C**

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i3k} \geq 25$$

$$H_{131} + H_{132} + H_{231} + H_{232} + H_{531} + H_{532} + H_{631} + H_{632} \geq 25$$

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i3k} \leq 35$$

$$H_{131} + H_{132} + H_{231} + H_{232} + H_{531} + H_{532} + H_{631} + H_{632} \leq 35$$

**4. House D**

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i4k} \geq 20$$

$$H_{141} + H_{142} + H_{241} + H_{242} + H_{541} + H_{542} + H_{641} + H_{642} \geq 20$$

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i4k} \leq 35$$

$$H_{141} + H_{142} + H_{241} + H_{242} + H_{541} + H_{542} + H_{641} + H_{642} \leq 35$$

**5. House E**

$$\sum_{i=1}^{4}\sum_{k=1}^{2} H_{i5k} \geq 10$$

$$H_{151} + H_{152} + H_{251} + H_{252} + H_{551} + H_{552} + H_{651} + H_{652} \geq 10$$

$$\sum_{i=1}^{4}\sum_{k=1}^{2} H_{i5k} \leq 25$$

$$H_{151} + H_{152} + H_{251} + H_{252} + H_{551} + H_{552} + H_{651} + \leq 25$$

#### 4.13.2.2 Budget Limit

$$\sum_{i=1}^{4}\sum_{j=1}^{5}\sum_{k}^{2} C_{ijk}\, H_{ijk} \leq B$$

**Expanded to**

$$73H_{111} + 58H_{112} + 88H_{121} + 70H_{122} + 97H_{131} + 81H_{132} + 122H_{141}$$
$$+ 96H_{142} + 152H_{151} + 120H_{152} + 75H_{211} + 60H_{212} + 88H_{221}$$
$$+ 70H_{222} + 98H_{231} + 80H_{232} + 116H_{241} + 96H_{242} + 150H_{251}$$
$$+ 118H_{252} + 75H_{511} + 61H_{512} + 89H_{521} + 71H_{522} + 96H_{531}$$
$$+ 77H_{532} + 120H_{541} + 95H_{542} + 153H_{551} + 120H_{552} + 74H_{611}$$
$$+ 58H_{612} + 92H_{621} + 70H_{622} + 102H_{631} + 83H_{632} + 127H_{641}$$
$$+ 99H_{642} 152H_{651} + 122H_{652} \leq 13000$$

#### 4.13.2.3 Project Duration

1. **First Sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{1jk}H_{1jk} \leq 1400$$

$$19.5H_{111} + 15H_{112} + 24H_{121} + 18.5H_{122} + 27.5H_{131} + 21H_{132} + 30H_{141} + 22.5H_{142} + 35H_{151} + 30H_{152} \leq 1400$$

2. **Second Sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{2jk}H_{2jk} \leq 1400$$

$$19H_{211} + 15.5H_{212} + 22.5H_{221} + 16.5H_{222} + 30H_{231} + 20H_{232} + 32.5H_{241} + 20.5H_{242} + 37.5H_{251} + 31H_{252} \leq 1400$$

3. **Fifth Sub-contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{5jk}H_{5jk} \leq 1400$$

$$45H_{511} + 30H_{512} + 52H_{521} + 40H_{522} + 56H_{531} + 40H_{532} + 64H_{541} + 40H_{542} + 77H_{551} + 56H_{552} \leq 1400$$

4. **Sixth Sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{6jk}H_{6jk} \leq 1400$$

$$45H_{611} + 35H_{612} + 48H_{621} + 38H_{622} + 62H_{631} + 43H_{632} + 63H_{641} + 54H_{642} + 77H_{651} + 56H_{652} \leq 1400$$

$H_{ijk} \geq 0$ *and integer*

**5. For all sub-Contractors**

$$\sum_{i=1}^{6}\sum_{j=1}^{5}\sum_{k=1}^{2} H_{ijk} = 170$$

$$H_{111} + H_{112} + H_{121} + H_{122} + H_{131} + H_{132} + H_{141} + H_{142} + H_{151} + H_{152} + H_{211} + H_{212} + H_{221} + H_{222} + H_{231} + H_{232} + H_{241} + H_{242} + H_{251} + H_{252} + H_{511} + H_{512} + H_{521} + H_{522} + H_{531} + H_{532} + H_{541} + H_{542} + H_{551} + H_{552} + H_{611} + H_{612} + H_{621} + H_{622} + H_{631} + H_{632} + H_{641} + H_{642} + H_{651} + H_{652} = 170$$

$H_{ijk} \geq 0$ *and integer*

## 4.14 Result Discussion Case Study (5)

After importing special information of mathematical model to the program, the outcome gotten was presented in table (4-11). The results revealed that the ideal distribution for sub-contractors was done in a way different from previously studied cases and outlined as follows:

1. No sub-contractor was excluded.
2. The biggest share of the cost was that of the first sub-contractor.
3. The estimated cost for project completion was (13.35) M$ which was the least among all cost cases, time period for project completion was (1207) days.

Table 4-11: Model Solution Case Study 5

| Sub-Contractor | No. of House $H_{ijk}$ | | | | | | | | | | Each sub-Contractor | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | | B | | C | | D | | E | | | |
| | FD | SD | FD | SD | FD | SD | FD | SD | FD | SD | Cost M $ | Duration (day) |
| 1st | 0 | 42 | 0 | 19 | 0 | 0 | 0 | 10 | 0 | 0 | 4.726 | 1207 |
| 2nd | 0 | 0 | 0 | 24 | 0 | 0 | 0 | 5 | 0 | 10 | 3.340 | 809 |
| 5th | 0 | 0 | 0 | 0 | 0 | 25 | 0 | 5 | 0 | 0 | 2.4 | 1200 |
| 6th | 0 | 18 | 0 | 12 | 0 | 0 | 0 | 0 | 0 | 0 | 1.884 | 1086 |
| Total of House | 60 | | 55 | | 25 | | 20 | | 10 | | - | |
| Project Cost M $ | 12.350 | | | | | | | | | | | |
| Duration (day) | 1207 | | | | | | | | | | | |

## 4.15 Case Study (6)

In this case study, constraints of available resources related to time needed for project completion was changed to become (1200) days.

These are presented in the following mathematical model:

### 4.15.1 Objective Function

$$Min\ Z = \sum_{i=1}^{4} \sum_{j=1}^{5} \sum_{k=1}^{2} C_{ijk}\, K_{ijk}$$

**Expanded to**

$$Min\ Z = 73H_{111} + 58H_{112} + 88H_{121} + 70H_{122} + 97H_{131} + 81H_{132} + 122H_{141} + 96H_{142} + 152H_{151} + 120H_{152} + 75H_{211} + 60H_{212} + 88H_{221} + 70H_{222} + 98H_{231} + 80H_{232} + 116H_{241} + 96H_{242} + 150H_{251} + 118H_{252} + 75H_{511} + 61H_{512} + 89H_{521} + 71H_{522} + 96H_{531} + 77H_{532} + 120H_{541} + 95H_{542} + 153H_{551} + 120H_{552} + 74H_{611} + 58H_{612} + 92H_{621} + 70H_{622} + 102H_{631} + 83H_{632} + 127H_{641} + 99H_{642} + 152H_{651} + 122H_{652}$$

### 4.15.2 Constraints

#### 4.15.2.1 Number of Houses

The total number of houses required (170), for each type according Table (4-12).

Table 4-12: Number of Houses

| House Type | A | B | C | D | E |
|---|---|---|---|---|---|
| Less Than or Equal | 60 | 60 | 35 | 35 | 25 |
| More Than or Equal | 40 | 40 | 25 | 20 | 10 |

1. **House A**

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i1k} \geq 40$$

$$H_{111} + H_{112} + H_{211} + H_{212} + H_{511} + H_{512} + H_{611} + H_{612} \geq 40$$

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i1k} \leq 60$$

$$H_{111} + H_{112} + H_{211} + H_{212} + H_{511} + H_{512} + H_{611} + H_{612} \leq 60$$

2. **House B**

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i2k} \geq 40$$

$$H_{121} + H_{122} + H_{221} + H_{222} + H_{521} + H_{522} + H_{621} + H_{622} \geq 40$$

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i2k} \leq 60$$

$$H_{121} + H_{122} + H_{221} + H_{222} + H_{521} + H_{522} + H_{621} + H_{622} \leq 60$$

### 3. House C

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i3k} \geq 25$$

$$H_{131} + H_{132} + H_{231} + H_{232} + H_{531} + H_{532} + H_{631} + H_{632} \geq 25$$

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i3k} \leq 35$$

$$H_{131} + H_{132} + H_{231} + H_{232} + H_{531} + H_{532} + H_{631} + H_{632} \leq 35$$

### 4. House D

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i4k} \geq 20$$

$$H_{141} + H_{142} + H_{241} + H_{242} + H_{541} + H_{542} + H_{641} + H_{642} \geq 20$$

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i4k} \leq 35$$

$$H_{141} + H_{142} + H_{241} + H_{242} + H_{541} + H_{542} + H_{641} + H_{642} \leq 35$$

5. **House E**

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i5k} \geq 10$$

$$H_{151} + H_{152} + H_{251} + H_{252} + H_{551} + H_{552} + H_{651} + H_{652} \geq 10$$

$$\sum_{i=1}^{4} \sum_{k=1}^{2} H_{i5k} \leq 25$$

$$H_{151} + H_{152} + H_{251} + H_{252} + H_{551} + H_{552} + H_{651} + \leq 25$$

#### 4.15.2.2 Budget Limit

$$\sum_{i=1}^{4} \sum_{j=1}^{5} \sum_{k}^{2} C_{ijk} H_{ijk} \leq B$$

**Expanded to**

$$73H_{111} + 58H_{112} + 88H_{121} + 70H_{122} + 97H_{131} + 81H_{132} + 122H_{141}$$
$$+ 96H_{142} + 152H_{151} + 120H_{152} + 75H_{211} + 60H_{212} + 88H_{221}$$
$$+ 70H_{222} + 98H_{231} + 80H_{232} + 116H_{241} + 96H_{242} + 150H_{251}$$
$$+ 118H_{252} + 75H_{511} + 61H_{512} + 89H_{521} + 71H_{522} + 96H_{531}$$
$$+ 77H_{532} + 120H_{541} + 95H_{542} + 153H_{551} + 120H_{552} + 74H_{611}$$
$$+ 58H_{612} + 92H_{621} + 70H_{622} + 102H_{631} + 83H_{632} + 127H_{641}$$
$$+ 99H_{642} 152H_{651} + 122H_{652} \leq 13000$$

#### 4.15.2.3 Project Duration

1. **First Sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{1jk}H_{1jk} \leq 1200$$

$$19.5H_{111} + 15H_{112} + 24H_{121} + 18.5H_{122} + 27.5H_{131} + 21H_{132} + 30H_{141} + 22.5H_{142} + 35H_{151} + 30H_{152} \leq 1200$$

2. **Second Sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{2jk}H_{2jk} \leq 1200$$

$$19H_{211} + 15.5H_{212} + 22.5H_{221} + 16.5H_{222} + 30H_{231} + 20H_{232} + 32.5H_{241} + 20.5H_{242} + 37.5H_{251} + 31H_{252} \leq 1200$$

3. **Fifth Sub-contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{5jk}H_{5jk} \leq 1200$$

$$45H_{511} + 30H_{512} + 52H_{521} + 40H_{522} + 56H_{531} + 40H_{532} + 64H_{541} + 40H_{542} + 77H_{551} + 56H_{552} \leq 1200$$

4. **Sixth Sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{6jk}H_{6jk} \leq 1200$$

$$45H_{611} + 35H_{612} + 48H_{621} + 38H_{622} + 62H_{631} + 43H_{632} + 63H_{641} + 54H_{642} + 77H_{651} + 56H_{652} \leq 1200$$

$H_{ijk} \geq 0$ *and integer*

**5. For all sub-Contractors**

$$\sum_{i=1}^{6}\sum_{j=1}^{5}\sum_{k=1}^{2} H_{ijk} = 170$$

$$H_{111} + H_{112} + H_{121} + H_{122} + H_{131} + H_{132} + H_{141} + H_{142} + H_{151} + H_{152} + H_{211} + H_{212} + H_{221} + H_{222} + H_{231} + H_{232} + H_{241} + H_{242} + H_{251} + H_{252} + H_{511} + H_{512} + H_{521} + H_{522} + H_{531} + H_{532} + H_{541} + H_{542} + H_{551} + H_{552} + H_{611} + H_{612} + H_{621} + H_{622} + H_{631} + H_{632} + H_{641} + H_{642} + H_{651} + H_{652} = 170$$

$H_{ijk} \geq 0$ *and integer*

## 4.16 Results Discussion of Case Study (6)

Table (4-13) presents outcomes of the mathematical model showing an ideal distribution different from all previously studied cases outlined as follows:

1. The total cost (12.350 M$) for project implementation was the least among all case studies, but equals that of case study (5).
2. (1200) days for project implementation was the least, while in case study (5) it was (1207) days.
3. None of the sub-contractors was excluded, but the biggest share was that of the sub-contractor.

4. Based on the results presented, case study (6) might be the ideal with regard to total cost and time needed for project implementation.

Table 4-13: Model Solution Case Study 6

| Sub-Contractor | No. of House $H_{ijk}$ | | | | | | | | | | Each sub-Contractor | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | | B | | C | | D | | E | | | |
| | FD | SD | FD | SD | FD | SD | FD | SD | FD | SD | Cost M $ | Duration (day) |
| 1st | 0 | 41 | 0 | 21 | 0 | 0 | 0 | 7 | 0 | 0 | 4.520 | 1161 |
| 2nd | 0 | 0 | 0 | 23 | 0 | 0 | 0 | 8 | 0 | 10 | 3.558 | 854 |
| 5th | 0 | 0 | 0 | 0 | 0 | 25 | 0 | 5 | 0 | 0 | 2.4 | 1200 |
| 6th | 0 | 19 | 0 | 11 | 0 | 0 | 0 | 0 | 0 | 0 | 1.872 | 1083 |
| Total of House | 60 | | 55 | | 25 | | 20 | | 10 | | - | |
| Project Cost M $ | 12.350 | | | | | | | | | | | |
| Duration (day) | 1200 | | | | | | | | | | | |

## 4.17 Case Study (7)

Results of mathematical models of previous studies revealed that houses with regard to their types and degree of implementation that were assigned to sub-contractors were (SD) because of the deviation of goal function to achieve the minimal cost

As the company assumed that 20% of citizens might wish to have houses of (FD) type, therefore a new mathematical model was designed based on the information added by the company as illustrated in Table (4-14).

Table 4-14: Number of Houses

| **House Type** | A-FD | A-SD | B-FD | B-SD | C-FD | C-SD | D-FD | D-SD | E-FD | E-SD |
|---|---|---|---|---|---|---|---|---|---|---|
| **Must be Equal** | 10 | 40 | 10 | 40 | 6 | 24 | 5 | 20 | 3 | 12 |

The mathematical model for case study (7) is as follow:

### 4.17.1 Objective Function

The research objective function was minimized cost of project, defined as follow:

$$Min\ Z = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{D} C_{ijk}\, H_{ijk}$$

**Expand to**

$$Min\ Z = 73H_{111} + 58H_{112} + 88H_{121} + 70H_{122} + 97H_{131} + 81H_{132} + 122H_{141} + 96H_{142} + 152H_{151} + 120H_{152} + 75H_{211} + 60H_{212} + 88H_{221} + 70H_{222} + 98H_{231} + 80H_{232} + 116H_{241} + 96H_{242} + 150H_{251} + 118H_{252} + 70H_{311} + 62H_{312} + 91H_{321} + 73H_{322} + 102H_{331} + 80H_{332} + 122H_{341} + 99H_{342} + 150H_{351} + 123H_{352} + 70H_{411} + 60H_{412} + 90H_{421} + 72H_{422} + 100H_{431} + 80H_{432} + 125H_{441} + 100H_{442} + 150H_{451} + 120H_{452} + 75H_{511} + 61H_{512} + 89H_{521} + 71H_{522} + 96H_{531} + 77H_{532} + 120H_{541} + 95H_{542} + 153H_{551} + 120H_{552} + 74H_{611} + 58H_{612} + 92H_{621} + 70H_{622} + 102H_{631} + 83H_{632} + 127H_{641} + 99H_{642} + 152H_{651} + 122H_{652}$$

### 4.17.2 Constraints

#### 4.17.2.1 Number of Houses

**1. House (A-FD)**

$$\sum_{i=1}^{6} H_{i11} = 10$$

$$H_{111} + H_{211} + H_{311} + H_{411} + H_{511} + H_{611} = 10$$

2. **House (A-SD)**

$$\sum_{i=1}^{6} H_{i12} = 40$$

$$H_{112} + H_{212} + H_{312} + H_{412} + H_{512} + H_{612} = 40$$

3. **House (B-FD)**

$$\sum_{i=1}^{6} H_{i21} = 10$$

$$H_{121} + H_{221} + H_{321} + H_{421} + H_{521} + H_{621} = 10$$

4. **House (B-SD)**

$$\sum_{i=1}^{6} H_{i22} = 40$$

$$H_{122} + H_{222} + H_{322} + H_{422} + H_{522} + H_{622} = 40$$

5. **House (C-FD)**

$$\sum_{i=1}^{6} H_{i31} = 6$$

$$H_{131} + H_{231} + H_{331} + H_{431} + H_{531} + H_{631} = 6$$

6. **House (C-SD)**

$$\sum_{i=1}^{6} H_{i32} = 24$$

$$H_{132} + H_{232} + H_{332} + H_{432} + H_{532} + H_{632} = 24$$

7. **House (D-FD)**

$$\sum_{i=1}^{6} H_{i41} = 5$$

$$H_{141} + H_{241} + H_{341} + H_{441} + H_{541} + H_{641} = 5$$

8. **House (D-SD)**

$$\sum_{i=1}^{6} H_{i42} = 20$$

$$H_{142} + H_{242} + H_{341} + H_{441} + H_{541} + H_{641} = 20$$

9. **House (E-FD)**

$$\sum_{i=1}^{6} H_{i51} = 3$$

$$H_{151} + H_{251} + H_{351} + H_{451} + H_{551} + H_{651} = 3$$

10. **House (E-SD)**

$$\sum_{i=1}^{6} H_{i52} = 12$$

$$H_{152} + H_{252} + H_{352} + H_{452} + H_{552} + H_{652} = 12$$

#### 4.17.2.2 Project Budget

The budget of the project was limited; (B) and B equal (13 Million) $.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k}^{D} C_{ijk} H_{ijk} \leq B$$

**Expanded to**

$$73H_{111} + 58H_{112} + 88H_{121} + 70H_{122} + 97H_{131} + 81H_{132} + 122H_{141} + 96H_{142} + 152H_{151} + 120H_{152} + 75H_{211} + 60H_{212} + 88H_{221} + 70H_{222} + 98H_{231} + 80H_{232} + 116H_{241} + 96H_{242} + 150H_{251} + 118H_{252} + 70H_{311} + 62H_{312} + 91H_{321} + 73H_{322} + 102H_{331} + 80H_{332} + 122H_{341} + 99H_{342} + 150H_{351} + 123H_{352} + 70H_{411} + 60H_{412} + 90H_{421} + 72H_{422} + 100H_{431} + 80H_{432} + 125H_{441} + 100H_{442} + 150H_{451} + 120H_{452} + 75H_{511} + 61H_{512} + 89H_{521} + 71H_{522} + 96H_{531} + 77H_{532} + 120H_{541} + 95H_{542} + 153H_{551} + 120H_{552} + 74H_{611} + 58H_{612} + 92H_{621} + 70H_{622} + 102H_{631} + 83H_{632} + 127H_{641} + 99H_{642} + 152H_{651} + 122H_{652} \leq 14000$$

#### 4.17.2.3 Project Duration

Duration for project less than or equal 1400 days.

1. **First sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{1jk}H_{1jk} \leq 1400$$

$$38H_{111} + 30H_{112} + 48H_{121} + 37H_{122} + 55H_{131} + 42H_{132} + 60H_{141} + 45H_{142} + 70H_{151} + 60H_{152} \leq 1400$$

2. **Second sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{2jk}H_{2jk} \leq 1400$$

$$38H_{211} + 31H_{212} + 45H_{221} + 33H_{222} + 60H_{231} + 40H_{232} + 65H_{241} + 41H_{242} + 75H_{251} + 62H_{252} \leq 1400$$

3. **Third sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{3jk}H_{3jk} \leq 1400$$

$$42H_{311} + 34H_{312} + 50H_{321} + 40H_{322} + 60H_{331} + 43H_{332} + 62H_{341} + 48H_{342} + 72H_{351} + 58H_{352} \leq 1400$$

4. **Fourth sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{4jk}H_{4jk} \leq 1400$$

$$40H_{411} + 35H_{412} + 54H_{421} + 40H_{422} + 60H_{431} + 40H_{432} + 60H_{441} + 44H_{442} + 72H_{451} + 45H_{452} \leq 1400$$

5. **Fifth sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{5jk}H_{5jk} \le 1400$$

$$45H_{511} + 30H_{512} + 52H_{521} + 40H_{522} + 56H_{531} + 40H_{532} + 64H_{541} + 40H_{542} + 77H_{551} + 56H_{552} \le 1400$$

6. **Sixth sub-Contractor**

$$\sum_{j=1}^{5}\sum_{k=1}^{2} t_{6jk}H_{6jk} \le 1400$$

$$45H_{611} + 35H_{612} + 48H_{621} + 38H_{622} + 62H_{631} + 43H_{632} + 63H_{641} + 54H_{642} + 77H_{651} + 56H_{652} \le 1400$$

$$H_{ijk} \ge 0 \; and \; integer$$

#### 4.17.2.4 Result Discussion of Case Study (7)

After the new available information of mathematical model was processed by the computer, the results were as follows:

1. There was no possible solution which indicates that meeting promises at one time was unlikely. Thus, having an ideal solution with a positive value was an imaginary variable and one of the resources might not meet all restrictions.
2. Based on paragraph (1), the budget set by the company was changed to be (14 M$) instead of (13 M$), thus providing a possible solution for the model.
3. Table (4-15) provided outcomes of the mathematical model of case study (7) whose results revealed that none of the sub-contractors was excluded. The total cost (13.526 $) was the biggest and the implementation period needed was (1400) days.

4. The results also present a better model distribution to include all types of houses with regard to areas and degree of implementation.

Table 4-15: Model Solution for Case Study 7

| Sub-Contractor | No. of House $H_{ijk}$ | | | | | | | | | | Each sub-Contractor | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | | B | | C | | D | | E | | | |
| | FD | SD | FD | SD | FD | SD | FD | SD | FD | SD | Cost M $ | Duration (day) |
| $1^{st}$ | 0 | 32 | 4 | 0 | 5 | 0 | 0 | 0 | 0 | 0 | 2.693 | 1372 |
| $2^{nd}$ | 0 | 0 | 0 | 10 | 1 | 0 | 2 | 9 | 0 | 8 | 2.838 | 1385 |
| $3^{rd}$ | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 0 | 3 | 0 | 0.816 | 402 |
| $4^{th}$ | 10 | 1 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 1.780 | 979 |
| $5^{th}$ | 0 | 0 | 0 | 0 | 0 | 24 | 0 | 11 | 0 | 0 | 2.893 | 1400 |
| $6^{th}$ | 0 | 7 | 0 | 30 | 0 | 0 | 0 | 0 | 0 | 0 | 2.506 | 1385 |
| Total of House | 10 | 40 | 10 | 40 | 6 | 24 | 5 | 20 | 3 | 12 | - | - |
| Project Cost M $ | 13.526 | | | | | | | | | | | |
| Duration (day) | 1400 | | | | | | | | | | | |

## 4.18 Comparison of the Results of All Cases

From the results of the seven study cases shown in Table (4-16), the following points are obvious:

1. The sixth study case was found to be the optimal one in terms of the lowest cost and the shortest time period required to finish the project. The cost was (12.350) Million US$ and the completion time period was (1200) days.
2. As illustrated in Figure (4-2), for all the study cases, the cost was within the pre-determined budget of 13M$, except for the seventh case, where, as a result of commitment to implement (20%) of the houses of (FD) type, there was a cost

overrun of (0.526) M$ and the pre-determined budget was raised accordingly to (14) M$.

3. For all the study cases, as shown in Figure (4-3), the time period required to accomplish the project ranged from (1200) to (1400) days.
4. According to each study case, it was obvious that the total cost amounted to (12.907) M$. Consequently, a price increase of more than

$$\frac{13-12.907}{13} \times 100\ \% = 0.72\ \%$$

would lead to budget insufficiency, which requires increasing the pre-determined budget; otherwise, the mathematical model will give an impossible solution. In the fifth and sixth study case, the allowed increase will be up to (5%) since the total cost amounted to (12.350) M$; where in the seventh study case , the allowed increase when the budget is (14) M$ reaches (3.38%) since the total cost was (13.526) M$. This reveals that cases five and six are optimal in terms of both cost reduction and resistance against inflation.

5. The total project completion cost for cases five and six was the same (12.35) M$, but the required time period for project completion in case six was less than that in case five by (7) days.
6. As for study cases one and two, compression of the estimated time period for project completion from (1400) days to (1200) days led to cost increase by (0.041) M$, which is consistent with the principles of engineering management, where time compression directly leads to cost increase.
7. Table (4-17) shows that the greatest share in terms of implementing the houses was for the second and fifth subcontractors for the majority of study cases, which indicates their suitable prices and time period estimated to accomplish the houses.

Table 4-16: Results for All Cases

| Case Study | House Type | | | | | | | | | | Duration Required (day) | Total Cost M $ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | | B | | C | | D | | E | | | |
| | FD | SD | FD | SD | FD | SD | FD | SD | FD | SD | Cost M $ | Duration (day) |
| 1 | 0 | 50 | 0 | 50 | 0 | 30 | 0 | 25 | 0 | 15 | 1400 | 12.907 |
| 2 | 0 | 60 | 0 | 55 | 0 | 25 | 0 | 20 | 0 | 10 | 1400 | 12.371 |
| 3 | 0 | 60 | 0 | 55 | 0 | 25 | 0 | 20 | 0 | 10 | 1200 | 12.421 |
| 4 | 0 | 50 | 0 | 50 | 0 | 30 | 0 | 25 | 0 | 15 | 1400 | 12.875 |
| 5 | 0 | 60 | 0 | 55 | 0 | 25 | 0 | 20 | 0 | 10 | 1207 | 12.350 |
| 6 | 0 | 60 | 0 | 55 | 0 | 25 | 0 | 20 | 0 | 10 | 1200 | 12.350 |
| 7 | 10 | 40 | 10 | 40 | 6 | 24 | 5 | 20 | 3 | 12 | 1400 | 13.526 |

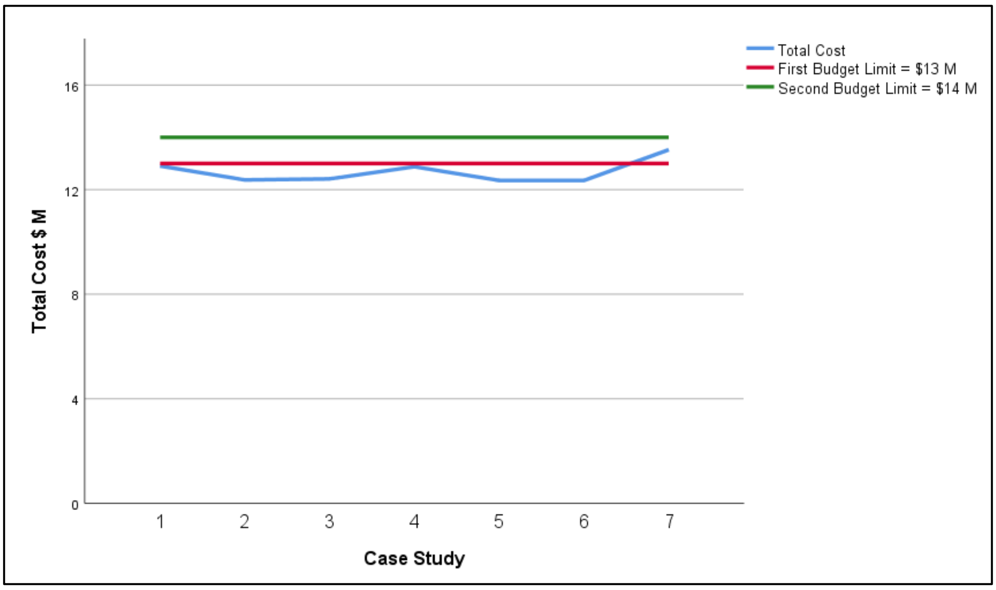


Figure 4-2: Total Cost for All Cases

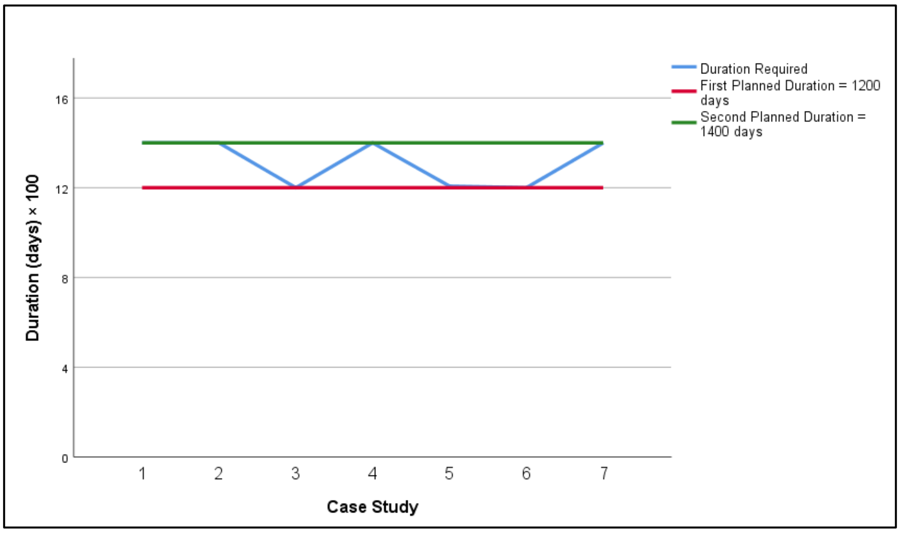

Figure 4-3: Time Required for All Cases

Table 4-17: Sub-contractor Total Cost for All Cases

| Sub-contractors | Case Study | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| | Total Cost | | | | | | |
| 1 | 2.668 | 2.778 | 2.320 | 3.796 | 4.726 | 4.520 | 2.693 |
| 2 | 3.088 | 2.940 | 2.520 | 4.410 | 3.340 | 3.558 | 2.838 |
| 3 | 0 | 0 | 0.880 | 0 | 0 | 0 | 0.816 |
| 4 | 0.705 | 1.400 | 1.952 | 0 | 0 | 0 | 1.780 |
| 5 | 2.785 | 2.893 | 2.670 | 2.785 | 2.400 | 2.400 | 2.893 |
| 6 | 2.554 | 2.360 | 2.070 | 1.884 | 1.884 | 1.872 | 2.506 |

# 5 Chapter Five: Conclusions and Recommendations

## 5.1 Conclusions

Several important conclusions could be made from the present study, as listed below:

1. The integer linear programming technique is an efficient method to implement the right decisions. Using the mathematical model, the number of units of house buildings were found to be referred to each sub-contractor.
2. The results found that application of the mathematical model in the decision was applicable.
3. Mathematical modeling is an efficient way to optimize the allocation of sub-contract on project.
4. The results indicated that the application of mathematical modeling is appropriate for every company or project when providing sufficient updated information.
5. The results gave the mathematical model for the seven study cases the project manager or the employer freely to choose the appropriate solution that meets the requirements of citizens for all the type of house.
6. It was evident from the model's outputs that it can find the lowest costs for houses according to their type, degree of implementation, and within the planned time and budget limitations.
7. The results of applying the mathematical models on seven study cases, it was indicated that an unbounded or no feasible region solution, except for the seventh study case, did not appear in the case of a solution that is not possible and was addressed by changing the available resources represented in the budget.

8. Based on the findings provided from the mathematical model for the first study case, the allowable inflation at prices was 0.72% and 4% for the mathematical model in the second study case, but excluding the two sub-contractors as in the sixth study case, the allowable inflation is up to 5%.
9. The findings indicated that when the company adheres to 20% of the houses according to their types, they will have a degree of implementation (FD), then the allocated budget for them is greater than other study cases.
10. The increase in inflation above the allowable values for each case is dealt by changing the available resources of the budget constraint within the mathematical model equations.
11. Throughout the results obtained from the seven academic cases:
    - A. The results indicated that the less expensive cost for the implementation of the project (12.350) M$ with a duration of (1200) days and within the mathematical model for the sixth case.
    - B. Within the seven study cases, the first sub-contractor with the largest share of the number of houses referred to it for implementation, which indicates low estimated prices, while the third sub-contractor was excluded because of its relatively high prices.
    - C. Doubling the teams working with the first and second sub-contractors and exclude the third and fourth sub-contractors can reduce the cost by (0.062) M$.
    - D. Changing the planned period of implementation of the project from (1400) days for the second study case to (1200) days for

the third study case, the total cost increased by (0.041) M$. This indicates that despite the similarities of all constraints, the process of crashing the planned period is below the increase of the cost.

E. Excluding the third and fourth sub- contractors in the case of the fifth study and in the sixth case, the total cost in the two cases is (12.350) M$, but the time required for implementation is (1207) days in the fifth and 1200 days in sixth case, and the reason is that the specific change of the constant type of houses changes from equal to greater or equal to smaller.

F. It is essential that the company manager and the owner of the project are in charge to take the decision to choose the solution that suits the company and the aspirations of citizens and the movement of the market.

G. Identifying 20% of the houses indicated that it should be ranked first in which the total cost has increased to become (13.526) M$ and the required necessary period (1400) days.

## 5.2 Recommendations

1. The necessity of providing sufficient information for each project in order to build a mathematical model for the problem at hand.
2. Using the integer linear programming to solve engineering problems that require the units to be integer numbers.
3. Encouraging companies to use mathematical modelling and optimization in the allocation of subcontractors.

4. Applications of linear programming and integer linear programming when planning cities, as represented (building new homes, building restoration, benefit planning, and public services).
5. Applying mathematical modelling when the restrictions imposed on projects are economic and in complex problems.
6. Companies should conduct a questionnaire on citizens ’desire for the type of houses, according to their area and degree of implementation, in order to build an accurate database of the mathematical model.
7. Conducting training for engineers through courses on modelling and optimization methods.
8. Marketing mathematical modeling to other companies through corporate communication and social media.

## 5.3 Future Work

1. Applying the linear or non-linear programming can be carried out by using other determinants such as labour, material and qualitative determinants.
2. Applying dynamic programming with the optimal allocation of contractors.

# References


- Abbas. M and Moula. M (2010), Solving Multiple Objective Integer Linear Programing, **Journal of the Italian Operation Research Society**, (29), 15-18.
- Abido,M.and A.M ElaZouni; (2010) , Multiobjectve evolutionary finance- based scheduling, Entire projects. Protolio. **Journal of Competing In Civil Engineering** 2010. 25(1), P 8597.
- Agrama, F. A. (2012). Multi-objective genetic optimization of linear construction projects. **HBRC Journal,** 8(2), 144-151.
- Azimi, M., Beheshti, R. R., Imanzadeh, M., & Nazari, Z. (2013). Optimal allocation of human resources by using linear programming in the beverage company. **Universal Journal of Management and Social Sciences**, *3*(5), 48-54.
- Basu, R., Aggarwal, R., & Jana, A. (2017). An Integrated Framework for Allocation of New Emergency Response Facilities in Developing Nations. **Process Integration and Optimization for Sustainability**, 1(2), 121-134.
- Bélanger, V., Ruiz, A., & Soriano, P. (2015). Recent advances in emergency medical services management. Faculté des sciences de l'administration, Université Laval., Sous le numéro FSA-2015-006.
- Billhard, H., Lujak, M., Sánchez-Brunete, V., Fernández, A., & Ossowski, S. (2014). Dynamic coordination of ambulances for emergency medical assistance services. **Knowledge-Based Systems**, *70*, 268-280.
- Chen, P. H., & Shahandashti, S. M. (2007, September). Simulated annealing algorithm for optimizing multi-project linear scheduling with multiple resource constraints. **In Proceedings of 24th International Symposium on Automation and Robotics in Construction (ISARC 2007), September (pp. 19-21).**

- Dantzig, G. (2016). **Linear programming and extensions**. Princeton university press.
- El. Abbasy. M, T Zayed and A. Elazouni "Finance-Based Scheduling for Multiple Project with Multi-Mode Activities". In **Construction Research Congress**, ASCE 2012.
- EL.kholy A.(2014) "A multi-objective fuzzy linear programming model for cash flow management' **International Journal of engineering Research and Application** ' vol 4 issue 8 , 152-163.
- Elazouni, A. M., & Gab-Allah, A. A. (2004). Finance-based scheduling of construction projects using integer programming. **Journal of Construction Engineering and Management**, 130(1), 15-24.
- El-Gafy, M. (2007). Resource Allocation for Repetitive Construction Schedules: An Ant Colony Optimization Approach. **In Proceedings of the ASC 43rd Annual International Conference, Flagstaff, Arizona**.
- ElGohary, Khaled, Remon Aziz and Hesham Abd-Elkhaled 2017 "Engineering Using ANN to Improve and Predict Construction Linear Productivity Under Different Influences, **Journal of Construction Engineering Management (ASCE)**. v (143).
- Ezema, B. I., & Amakom, U. (2012). Optimizing profit with the linear programming model: A focus on golden plastic industry limited, Enugu, Nigeria. **Interdisciplinary Journal of Research in Business**, 2(2), 37-49.
- Gadge, S. B., Gorantiwar, S. D., Kumar, V., & Kothari, M. (2014). Linear programming approach for allocation of land and water resources in canal command area under surface method of irrigation-A case study. **International**

**Journal of Innovative Research in Science, Engineering and Technology,** 3(4), 153- 168**.**

- Genova, K., & Guliashki, V. (2011). Linear integer programming methods and approaches-a survey. **Journal of Cybernetics and Information Technologies,** 11(1).
- GHADERI, K., SAMANI, M., Mousavi, S. J., Eslami, H. R., & Arab, D. R. (2010). Optimal operation modeling of reservoirs using mixed integer linear programming (MILP). **Iran-Water Resource Research**, 6(1), (15-27).
- Gherghina, R., & Duca, I. (2013). Using Linear Programming in order to optimize the Allocation of Resources for Investment. **Journal of Knowledge Management, Economics and Information Technology**, 3(1), 1-12.
- Griffin, P. M., Scherrer, C. R., & Swann, J. L. (2008). Optimization of community health center locations and service offerings with statistical need estimation. **IIEE Transactions on Engineering Management**, 40(9), 880-892.
- Hamdjatou Kane, Alban Tissier. A Resources Allocation Model for Multi-Project Management. **9th International Conference on Modeling, Optimization & SIMulation**, Jun 2012, Bordeaux, France. ffhal-00728599.
- Hamdy, A.Taha (2017) "**Operation Research; an Introduction**, 10th eduction, international.
- Hamzeh H AL-safadi, Rami A. Ibrahim A. (2018) optimal crashing time Scheduling for Mega projects '**International Journal of Management** ' v(9) issue 2 ,(2018).
- Jia, H., Ordóñez, F., & Dessouky, M. (2007). A modeling framework for facility location of medical services for large-scale emergencies. **IIEE Transactions**, 39(1), 41- 55.

- Leo liberti, (2010). "**Problems and Exercises in Operation Research,** École polytechnique".
- Li, X., Zhao, Z., Zhu, X., & Wyatt, T. (2011). Covering models and optimization techniques for emergency response facility location and planning: a review. **Mathematical Methods of Operations Research**, 74(3), 281-310.
- Matousek, Jiri, Gartner, Brend "**Understanding and Using Linear Programming**", published by springer-verlay Berlin Heidelberg, New York, USA, 2017.
- Meinzer, N., & Storandt, S. (2014, June). Decision Support in Emergency Medical Systems: New Strategies for Dynamic Ambulance Allocation. **In the First AAAI Workshop on World Wide Web and Public Health Intelligence** (W3PHI-2014).
- Mohamed Rasmy, Hesham A.E Abdelsalam ,and Rania R.Hussein(2008).Multi-objective Tim-cost Trade-off analysis in critical chain project Networks using pare to simulated Annealing "**INFOS 2008**" March 27-29 Cairo-Egypt ,Faculty of computer and information-cairo university.
- Nassar, K. (2011). Evolutionary optimization of resource allocation in repetitive construction schedules. **Journal of Information Technology in Construction (ITcon)**, 10(18), 265-273.
- Rajeiyan, K., Nejati, F. K., Hajati, R., Safari, H. R., & Alizadeh, E. (2013). Using Linear Programming in Solving the Problem of Services Company's Costs. **Singaporean Journal of Business, Economics and Management Studies**, 51(1112), 1-6.

- Shah, K. A., Farid, F., & Baugh, J. W. (1993). Optimal resource leveling using integer-linear programming. **In Computing in Civil and Building Engineering** (pp. 501- 508). ASCE.
- Sharma, J. K., & Sharma, J. K. (2004). **Operations Research Problems and Solutions**. Macmillan India Limited.
- Sherren Abunada , Ibrahim A.Mohammed (2018) optimization model in sub-contractors assignment in Jordan , **International Journal of Applied Engineering Research** ,vol(13) issue 1.
- Toro-DíAz, H., Mayorga, M. E., Chanta, S., & Mclay, L. A. (2013). Joint location and dispatching decisions for emergency medical services. **Computers & Industrial Engineering**, 64(4), 917-928.
- Williams, H Pual (2009). "**Model Building in Mathematical Programing**" 4$^{th}$ Edition, publisher: John Willy and Son., Ltd (UK).
- Zheng, D. X., Ng, S. T., & Kumaraswamy, M. M. (2004). Applying a genetic algorithm-based multi objective approach for time-cost optimization. **Journal of Construction Engineering and management,** 130(2), 168-176.
- Zheng, Z., & Zhang, H. (2016). A Mixed-Integer Linear Programming Scheduling Optimization Model for Refinery Production. **Chemical Engineering Transaction**, v (51).

# Appendix A

Computer program using MATLAB "Linprog" built- in function.

Note the program is varied according to the inputs of mathematical model, but the follow is some examples of cases study.

# Case$_1$



```
clear
clc
F= [73 58 88 70 97 81 122 96 152 120 75 60 88 70 98 80 116 96 150 118 70 62 91 73 102 80↙
122 99 150 123 70 60 90 72 100 80 125 100 150 120 75 61 89 71 96 77 120 95 153 120 74 58↙
92 70 102 83 127 99 152 122];
A=[39 30 48 37 55 42 60 45 70 60 zeros(1,50); zeros(1,10) 38 31 45 33 60 40 65 41 75 62↙
zeros(1,40); zeros(1,20) 42 34 50 40 60 43 62 48 72 58 zeros(1,30); zeros(1,30) 40 35 54↙
40 60 40 60 44 72 45 zeros(1,20); zeros(1,40) 45 30 52 40 56 40 64 40 77 56 zeros(1,10);↙
zeros(1,50) 45 35 48 38 62 43 63 45 77 56; 73 58 88 70 97 81 122 96 152 120 75 60 88 70↙
98 80 116 96 150 118 70 62 91 73 102 80 122 99 150 123 70 60 90 72 100 80 125 100 150↙
120 75 61 89 71 96 77 120 95 153 120 74 58 92 70 102 83 127 99 152 122];
B=[1400; 1400; 1400; 1400; 1400; 1400; 13000];
Aeq=[1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1↙
0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0; 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0↙
0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0; 0 0 0 0 1 1 0 0 0 0↙
0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0↙
1 1 0 0 0 0; 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1↙
0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0; 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0↙
0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1];
Beq=[50; 50; 30; 25; 15];
lb=[linspace(0,0,60)];
[x,fval] = linprog (F,A,B,Aeq,Beq,lb);
fval=round(fval)
y=(round(x))'
```

Columns 1 through 27

0 46 0 0 0 0 0 0 0 0 0 0 0 15 0 0 0 20 0
1 0 0 0 0 0 0 0

Columns 28 through 54

0 0 0 0 1 0 1 0 0 0 0 0 14 0 0 0 0 0 30
0 5 0 0 0 3 0 34

Columns 55 through 60

0 0 0 0 0 0

# Case$_4$



```
clear
clc
F= [73 58 88 70 97 81 122 96 152 120 75 60 88 70 98 80 116 96 150 118 75 61 89 71 96 77↙
120 95 153 120 74 58 92 70 102 83 127 99 152 122];
A=[19.5 15 24 18.5 27.5 21 30 22.5 35 30 zeros(1,30); zeros(1,10) 19 15.5 22.5 16.5 30↙
20 32.5 20.5 37.5 31 zeros(1,20); zeros(1,20) 45 30 52 40 56 40 64 40 77 56 zeros(1,10);↙
zeros(1,30) 45 35 48 38 62 43 63 45 77 56];
B=[1400; 1400; 1400; 1400; 13000];
Aeq=[1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 ; 0↙
0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0; 0 0 0 0 1↙
1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0; 0 0 0 0 0 0 1 1 0↙
0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0; 0 0 0 0 0 0 0 0 1 1 0 0 0↙
0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 173 58 88 70 97 81 122 96 152 120 75↙
60 88 70 98 80 116 96 150 118 75 61 89 71 96 77 120 95 153 120 74 58 92 70 102 83 127 99↙
152 122; ];
Beq=[50; 50; 30; 25; 15];
lb=[linspace(0,0,40)];
[x,fval] = linprog (F,A,B,Aeq,Beq,lb);
fval=round(fval)
y=(round(x))'
```

Columns 1 through 27

0 32 0 14 0 0 0 10 0 0 0 0 0 24 0 0 0 10 0
15 0 0 0 0 0 30 0

Columns 28 through 40

5 0 0 0 18 0 12 0 0 0 0 0 0

# $Case_6$



```
clear
clc
F= [73 58 88 70 97 81 122 96 152 120 75 60 88 70 98 80 116 96 150 118 75 61 89 71 96 77↙
120 95 153 120 74 58 92 70 102 83 127 99 152 122];
A=[19.5 15 24 18.5 27.5 21 30 22.5 35 30 zeros(1,30);
    zeros(1,10) 19 15.5 22.5 16.5 30 20 32.5 20.5 37.5 31 zeros(1,20);
    zeros(1,20) 45 30 52 40 56 40 64 40 77 56 zeros(1,10);
    zeros(1,30) 45 35 48 38 62 43 63 45 77 56;
    1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0;
    0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0;
    0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0;
    0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0;
    0 0 0 0 0 0 0 0 -1 -1 0 0 0 0 0 0 0 0 -1 -1 0 0 0 0 0 0 0 0 -1 -1 0 0 0 0 0 0 0 0 -1↙
-1;
    -1 -1 0 0 0 0 0 0 0 0 -1 -1 0 0 0 0 0 0 0 0 -1 -1 0 0 0 0 0 0 0 0 -1 -1 0 0 0 0 0 0↙
0 0;
    0 0 -1 -1 0 0 0 0 0 0 0 0 -1 -1 0 0 0 0 0 0 0 0 -1 -1 0 0 0 0 0 0 0 0 -1 -1 0 0 0 0↙
0 0;
    0 0 0 0 -1 -1 0 0 0 0 0 0 0 0 -1 -1 0 0 0 0 0 0 0 0 -1 -1 0 0 0 0 0 0 0 0 -1 -1 0 0↙
0 0;
    0 0 0 0 0 0 -1 -1 0 0 0 0 0 0 0 0 -1 -1 0 0 0 0 0 0 0 0 -1 -1 0 0 0 0 0 0 0 0 -1 -1↙
0 0;
    0 0 0 0 0 0 0 0 -1 -1 0 0 0 0 0 0 0 0 -1 -1 0 0 0 0 0 0 0 0 -1 -1 0 0 0 0 0 0 0 0 -1↙
-1; 73 58 88 70 97 81 122 96 152 120 75 60 88 70 98 80 116 96 150 118 75 61 89 71 96 77↙
120 95 153 120 74 58 92 70 102 83 127 99 152 122];
B=[1200; 1200; 1200; 1200; 60; 60; 40; 35; 25; -40; -40; -25; -20; -10; 13000];
Aeq=ones(1,40);
Beq=[170];
lb=zeros(1,40);
[x,fval] = linprog (F,A,B,Aeq,Beq,lb);
fval=round(fval)
y=round(x)'
```

Columns 1 through 27

0 41 0 21 0 0 0 7 0 0 0 0 0 23 0 0 0 8 0
10 0 0 0 0 0 25 0

Columns 28 through 40

5 0 0 0 19 0 11 0 0 0 0 0 0

# البرمجة الخطية الصحيحة في تطوير مشاريع التجديد الحضري
# (العراق – حالة دراسية)

**إعداد:**

**معتصم ابراهيم عبد**

**المشرف:**

**الدكتور سفيان هياجنة**

**الملخص**

بعد احتلال العراق عام ٢٠٠٣ وما تعرض له ونظرا للظروف الإقتصادية الراهنة أصبحت ضرورة الإستغلال الأمثل للمواد حاجة ملحة.

حيث تهدف الدراسة الى استخدام البرمجة الخطية الصحيحة من خلال النمذجة الرياضية في التطوير الحضري والخاص ببناء منازل جديدة كحالة دراسية في العراق.

بُني نموذج رياضي للتوزيع الأمثل للمقاولين الثانويين لتنفيذ (١٧٠) منزل مقترح مختلف المساحات (١٥٠، ١٨٠، ٢٠٠، ٢٥٠، ٣٠٠) متر مربع وبمستويين للتنفيذ (درجة اولى، درجة ثانية) لهدف تحقيق ادنى التكالف ضمن المدة المخططة.

جُمعت المعلومات من خلال عطاءات ستة من المقاولين الثانويين والمتضمنة سعر الوحدة الواحدة حسب النوع ودرجة التنفيذ وكذلك المدة اللازمة لإنجاز الوحدة الواحدة.

تم بناء سبع نماذج رياضية لسبع حالات دراسية لتحقيق تحليل الحساسية ومن خلال تغير الموارد المتاحة والمتطلبات، تغير شكل المحددات، استبعاد بعض متغيرات القرار والمحددات الخاصة بالمنافسة واضافة قيود جديدة.

وظفت البرمجة الخطية الصحيحة في حل معادلات النموذج الرياضي بمساعدة برنامج (LIN Prog)

اظهرت النتائج امكانية تطبيق النمذجة الرياضية والبرمجة الخطية الصحيحة بفعالية لتحقيق ادنى كلف للتنفيذ وحسب المدة المخططة، وتخصيص أمثل للمقاولين الثانويين.

كما اظهرت امكانية مدير المشروع او صاحب العمل اختيار الحل الامثل وحسب متطلبات الشركة.

توصي الدراسة بضرورة التوسع باستخدام النمذجة الرياضية لحل المشاكل في كثير من المشاريع الإنشائية.

**الكلمات الدالّة**: المشاريع الإنشائية، النمذجة الرياضية، البرمجة الخطية الصحيحة، الأمثلية.